2007-04-01

What’s So Hard About Algebra? A Grounded Theory Study of Adult Algebra Learners

Michael Steele Reese EdD
University of San Diego

WHAT'S SO HARD ABOUT ALGEBRA?

A GROUNDED THEORY STUDY OF ADULT ALGEBRA LEARNERS

---

by

Michael Steele Reese

A Dissertation Submitted to the Faculty of

San Diego State University and the University of San Diego

in Partial Fulfillment

of the Requirements for the Degree

Doctor of Education

Dissertation Committee:

Janet Bowers, Ph.D., San Diego State University
Nancy Farnan, Ph.D., San Diego State University
Noriyuki Inoue, Ph.D., University of San Diego

April 2007

DEDICATION

This dissertation is dedicated to Leola, Miguel, and Naomi, the three loves of my life.

El mundo para ellos no es un concurso de objetos en el espacio; es una serie heterogénea de actos independientes. Es sucesivo, temporal, no espacial. No hay sustantivos en la conjetural *Ursprache* de Tlön, de la que proceden los idiomas "actuales" y los dialectos: hay verbos impersonales, calificados por sufijos (o prefijos) monosilábicos de valor adverbial. Por ejemplo: no hay palabra que corresponda a la palabra *luna*, pero hay un verbo que sería en español *lunecer* o *lunar*. *Surgió la luna sobre el río* se dice *hlör u fang axaxaxas mlö* o sea en su orden: hacia arriba (*upward*) detrás duradero-fluir luneció. (Xul Solar traduce con brevedad: upa tras perfluyue lunó. *Upward, behind the onstreaming it mooned.*)

— Jorge Luis Borges
Tlön, Uqbar, Orbis Tertius

ABSTRACT OF THE DISSERTATION

What's So Hard About Algebra?
A Grounded Theory Study of Adult Algebra Learners
by
Michael Steele Reese
Doctor of Education in Educational Technology
San Diego State University and the University of San Diego, 2007

In California community colleges, fewer than half of students who enroll in basic algebra courses finish with a grade of C or better. Such a low success rate creates an intense demand on institutional resources, including faculty efforts, tutorial services, classroom availability, and financial aid. Furthermore, students who do poorly in algebra also tend to struggle in other quantitative courses. While research suggests that child algebra learners tend to exhibit specific misconceptions, not much is known about misconceptions held by adult algebra learners. Research does indicate, however, that certain general learning characteristics are common among adult learners. The present study employed a grounded theory approach to examine (1) what pedagogical factors influence adult algebra learning, (2) whether adult algebra learners have similar misconceptions to those held by children, and (3) how necessary it is to consider general adult learning characteristics in developing curricula. Data were acquired through clinical interviews of adult community college students. The only criteria for inclusion in the study were that the participant be at least 18 years of age and currently enrolled in beginning or intermediate algebra at San Diego Mesa College. Findings were that: (1) certain pedagogical factors influence adult algebra learning, including instructional style and policies, course activities, learning aids, and course pacing; (2) participants demonstrated some of the same misconceptions as those held by children when learning algebra, including the *letter as label* and *graph as path* misconceptions, but not the *expression as procedure* misconception; (3) participants indicated that they learn algebra more successfully when general adult learning characteristics guide curriculum development, such as those specified by Knowles' theory of andragogy, academic fossilization, metacognition, and several unanticipated adult characteristics revealed by this study; and (4) very few significant differences emerged among genders, ethnic groups, or age levels, supporting the generalizability of the findings. A unique feature of this study is the open-ended form of data that were collected. Participants freely generated categories themselves rather than answering specific, previously designed questions. Publications from this study will benefit community college students by making faculty more aware of difficulties encountered by adult students during the learning of algebra, thereby putting them in a better position to develop and implement curricula and institute techniques that address such difficulties. Another potential benefit of this study is that select students were able to anonymously voice thoughts and opinions regarding the teaching and learning of algebra in community colleges.

TABLE OF CONTENTS

LIST OF TABLES

PAGE

LIST OF FIGURES

ACKNOWLEDGEMENTS

I owe my advisor, Janet Bowers, so much for her prodding, patience, and insight, without which I may never have completed this dissertation. She has been a wonderful instructor, mentor, and friend throughout the entire process. Thanks, Janet.

I am so grateful to Nancy Farnan and Noriyuki Inoue for serving on my committee, reading my drafts, offering excellent ideas and leads, and having lots of patience with me. Nancy, thanks for teaching me about grounded theory. Qualitative methods with you was one of my favorite doctoral courses. Janet, Nancy, and Nori, I cannot imagine having a better committee than you three.

My doctoral program has been extremely supportive of me. I thank all the faculty and administrators in the program for what they have taught me and for the motivation they have given me to finish my degree. Beth and Caroline, you two are terrific.

The most important ingredient in my study is the students themselves. I thank the 29 adult algebra students at Mesa College who were courageous enough to spend an hour baring their mathematical souls to me. I could not have done this without you.

My parents and sisters have always supported me in my endeavors. I appreciate everything they have done for me. Thanks especially to you, Mom, for devoting so much of your time to my wife, my children, and my home so that I could finish this dissertation.

Finally, I thank my wife, Leola, my son, Miguel, and my daughter, Naomi, for their love, patience, care, and playfulness during my doctoral program and always. I know how lucky I am to have you.

CHAPTER 1

INTRODUCTION

Some people think that it is not important for children to learn algebra (Cohen, 2006). In fact, the majority of U.S. parents believe that the amount of mathematics instruction their children receive in school is enough (Johnson, Arumi, Ott, & Remaley, 2006) and that their children's future jobs will not require math or science. Indeed, almost half of those children claim they would be very unhappy if their future employment did involve math or science. This is in sharp contrast to what those who actually hire employees are saying (Gouvêa, 2006). Business leaders and heads of technology- and science-based industries require their employees to be skilled in algebra (Eisenbud, 2004; NCTM, 2000). Most high-paying industries today have a dependence on algebra, causing the subject to act as a virtual gatekeeper to higher socioeconomic levels. Because of this, some consider the learning of algebra a civil rights issue (Moses, 1995).

Unfortunately, students today are failing algebra at unprecedented rates. This is happening not once, but twice: first in secondary school, and again in college. National reports have highlighted this problem (Braswell et al., 2001; Mullis, Martin, Gonzalez, & Chrostowski, 2004; National Commission on Excellence in Education, 1983), extensive funding has been provided for its study (Gertsen, 2004), and legislation has been passed in an attempt to rectify it (United States Congress, 2002). Most research on algebra learning has examined children rather than adults; however, over 700,000 college students took beginning and intermediate algebra courses in 2000 (Lutzer, Maxwell, & Rodi, 2006). Such a large number of adults taking algebra today warrants more research on adult algebra learners. This qualitative study explores adult algebra learning in a novel way, namely, by asking adult students themselves what is difficult about learning algebra.

BACKGROUND

Algebra is a prerequisite subject for most quantitative academic fields. Students are expected to have mastery of algebraic skills before they take science, engineering, business, or higher-level mathematics courses. Figure 1 illustrates interdependencies among various quantitative fields. In the figure, a block supported by another indicates a dependence of the upper field on the lower. Note that algebra lies at the foundation of the entire structure, i.e., every field in the figure depends on algebra.

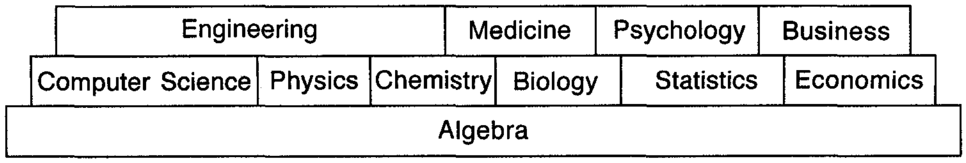

Figure 1. Interdependence of quantitative fields.

In 2000, the U.S. Department of Education's national report card for mathematics (Braswell et al., 2001) revealed that 34% of eighth-grade students performed below the basic skill level in mathematics, and only 27% performed at or above the proficient level. At the college level, studies have reported that 40% to 60% of algebra students either fail, drop, or withdraw each term (Small, 2006). California's SB 1354 law (Poochigian, 2000) requires all secondary students to complete a course in Algebra I; even so, nearly half of freshmen entering the California State University system must take remedial courses in algebra (Poochigian, 2004). At San Diego Mesa College, over 1500 students register for remedial algebra courses each semester, yet only about one third of them are successful in those courses (SDCCD IRP, 2004), where success is defined as achieving a course grade of A, B, or C (see Figure 2). Mesa College is not unusual; success rates in remedial algebra courses are similar at colleges and universities across the United States (Mullis, Dossey, Owen, & Phillips, 1991).

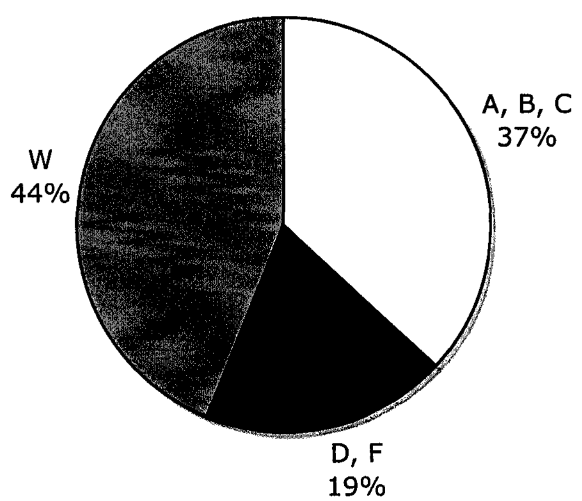

Figure 2. Mesa College algebra grades, spring 1999 to fall 2000 ($N = 6146$).

Poor algebra performance in the U.S. may have both individual and societal economic consequences. For example, only about 70,000 engineers graduated from U.S. universities last year, compared to almost a million from universities in China and India (Kra & Robinson, 2006). Besides the longer-term consequence of curtailing individuals' prospects of being employed in technical industries and business, the low success rate in algebra may have several immediate negative consequences for students and schools. Students may suffer financially, emotionally, and psychologically when they fail and are required to repeat courses (Preis & Biggs, 2001). Because algebra is often a prerequisite for other courses, some of which may be offered only occasionally, a student who fails algebra may delay his or her progress towards a degree for a year or more. Students who fail algebra twice may not be allowed to take the course a third time; hence, they may be forced to change majors or drop out of college altogether. Even when students eventually pass algebra, many do not learn it well and suffer in subsequent quantitative courses as a result (Ocken, 2004). The large number of students repeating courses places a drain on college resources (Toubassi, 1999). Faculty may become dispirited when they see the same students in their algebra classes semester after semester. Often there are not enough faculty or classrooms to teach higher-level mathematics courses because of the great demand for remedial courses. The large number of students who repeat courses may necessitate increased levels of tutorial services, financial aid, and other campus resources.

Statement of the Problem

The assumption underlying this study is that the better educators understand factors influencing adult student success in learning algebra, the better they can design curriculum and instruction to serve the students effectively. This is not an unreasonable assumption. Teachers have reported that the most important element in the design of instruction is information about students (Borko & Shavelson, 1990). The goals of the study are (1) to develop a theory of adult algebra learning, grounded in substantive data, that describes pedagogical factors that influence the learning of algebra by adults, (2) to determine whether misconceptions held by child algebra learners are also held by adults when they learn algebra, and (3) to examine the influence that general adult learning characteristics have on the learning of algebra by adults. The following research questions derive from these goals.

1. What pedagogical factors significantly influence the learning of algebra by adults?
2. Do adult algebra learners hold misconceptions about algebra that are similar to common misconceptions held by children who are learning algebra?
3. How influential are general adult learning characteristics when it comes to adults learning algebra?

This study is explorational and qualitative in nature. The primary product of the study is a theory grounded in substantive data about difficulties facing adult algebra learners.

PURPOSE OF THE STUDY

A large body of research exists on child algebra learners (Booth, 1988; Ginsburg, 1975; Henningsen & Stein, 1997; Kieran, 1979; Kuchemann, 1981; Mullis et al., 2004; Nathan & Koedinger, 2000; NCTM, 2000; Petitto, 1979; Rosnick, 1981; Rosnick & Clement, 1980; Sfard, 1991; Tall et al., 2000). We know, for example, that children tend to exhibit specific types of errors and share common misconceptions when learning algebra. Children often have poor number sense and lack understanding of variables and algebraic conventions. We do not have similar knowledge regarding adults due to the scarcity of research on adult algebra learners.

As is the case with research on children, research on adult algebra learners often implicitly assumes that the impediments to adult algebra learning lie within the domain of control of the learner. For instance, one pre/post-test study (Greenberg, 1991) of a central Texas community college examined variables such as mathematics enjoyment, amount of assistance needed to understand the course material, belief of personal ability in mathematics, and course load. An ongoing study of student success in algebra in California community colleges (Harrison & Teegarden, 2004) considers a host of personal, environmental, and historical factors to be the main contributors to college students' lack of success in algebra. These factors include personal life issues, financial concerns, lack of essential skills such as reading, arithmetic, time management and study skills, math or test anxiety, and lack of motivation or interest. Variables considered in a study of community colleges in Virginia (Waycaster, 2001) included course load, attendance, and class participation rates.

Each of the above studies exhibits the tacit assumption that factors influencing adult student success and failure in algebra lie within the domain of control of the student. Unfortunately, many of these student-centered factors have limited potential for short-term remediation at the college level. As valiant as their efforts are, colleges can do only so much to help unprepared students learn to read, study, manage time and money, overcome test and math anxieties, and appreciate mathematics. On the other hand, there may exist factors influencing success in algebra that are centered in institutions or faculty and thus are much more controllable. For instance, it is not hard to imagine that instructional delivery methods, classroom policies, subject content, instructor attitudes, or other instructor-centered factors could have an effect on how well adult students learn algebra. Many of these factors can be manipulated by instructors, but without an awareness of their existence or their effects on student learning, instructors may not manipulate them in such a way as to most benefit their students. In order to combat the high failure rate among adult algebra learners, it is important to examine what factors external to the adult student's personal domain of control may have an influence on the learning of algebra.

The purpose of this study is to investigate, in an unbiased and open-minded fashion, what factors influence adult learning of algebra. The study does not focus on student-centered factors, such as quality of childhood schooling or socioeconomic status, since colleges can do little in the short term to counter them. Instead, it focuses on instructor- and institution-centered factors, i.e., pedagogical factors that have a significant influence on adult algebra learning, where pedagogy is defined as the art and science of helping people learn, including the actions, objects, and attitudes under instructor or institutional control during the teaching and learning process. Informants in this study are the adult algebra learners themselves. The study thereby gives anonymous public voice to representative students so that they can describe in their own terms why they may be struggling with learning and what pedagogical factors they think influence their success. The study also gives participants an opportunity to reflect on their own learning and attitudes towards instruction and learning.

THEORETICAL BASES AND ORGANIZATION

Chapter 3 describes the grounded theory approach this study incorporates for collecting and analyzing data. As in other grounded theory research, the study produces a theory substantially derived from data. Nevertheless, in order to provide a theoretical framework, the study establishes the following research hypotheses.

1. Certain pedagogical factors at Mesa College have a significant influence on the learning of algebra by many adult students.
2. Many adult students have similar misconceptions when they learn algebra to those held by children who are learning algebra. Specifically, they exhibit the following misconceptions: *expression as procedure*, *letter as label*, and *graph as path.*
3. Adults learn algebra more successfully when general adult learning characteristics guide curriculum development. These characteristics include the five specified by Knowles in his theory of andragogy—self-concept, experience, readiness to learn, time, and orientation to learning—as well as academic fossilization and metacognition.

Figure 3 illustrates the various theories informing this study and the factors affecting learning that were expected to be uncovered by the study. These are discussed in the remainder of this section and in Chapter 2.

As the first hypothesis of this study states, the study explicitly assumes the existence of pedagogical factors influencing the learning of algebra by adults. Figure 4 was designed to illustrate the teaching-learning process. Notice that the student symbolically lies at the center of the figure. In the figure, a student interacts with four distinct entities during learning: ego, media, peers, and the instructor. Students interact with their egos when they reflect on what is being learned, why it is being learned, or under what conditions the learning may be useful. Further student-ego interactions occur when students consciously monitor or regulate their

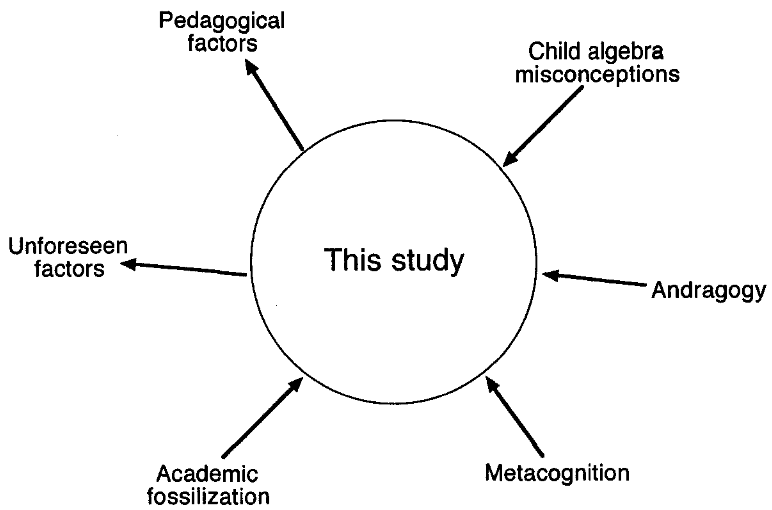

Figure 3. Theories informing this study and factors affecting learning that the study uncovers.

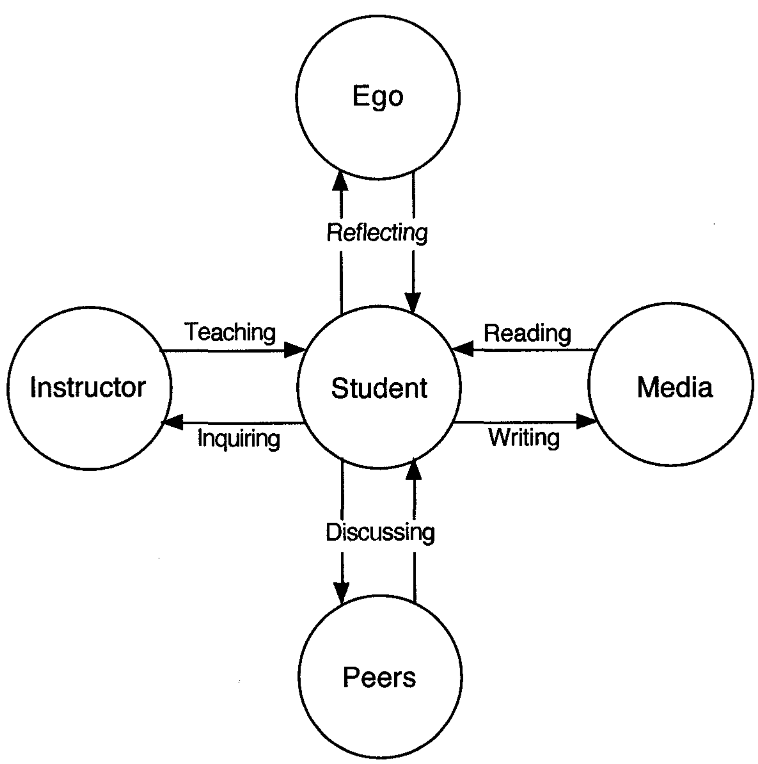

Figure 4. The teaching-learning process.

own learning in order to make it more effective or efficient. Student-peer interactions occur when students meet to discuss what is being learned. During such interactions, they may teach each other, test each other's knowledge, hypothesize together, or try to understand material together. Students interact with media through reading the textbook, observing videotaped lectures, writing homework problems, or taking online or written practice exams. Finally, the most visible interactions in the teaching-learning process occur between the instructor and the students during classroom time. Teachers lecture, give exams, go over homework solutions, and conduct a variety of classroom activities, while students listen, take notes, and ask questions to clarify concepts.

Of course, if the teaching-learning process worked well in all cases, there would be no need for this study. Sadly, the process often breaks down, as Figure 5 illustrates. Students may

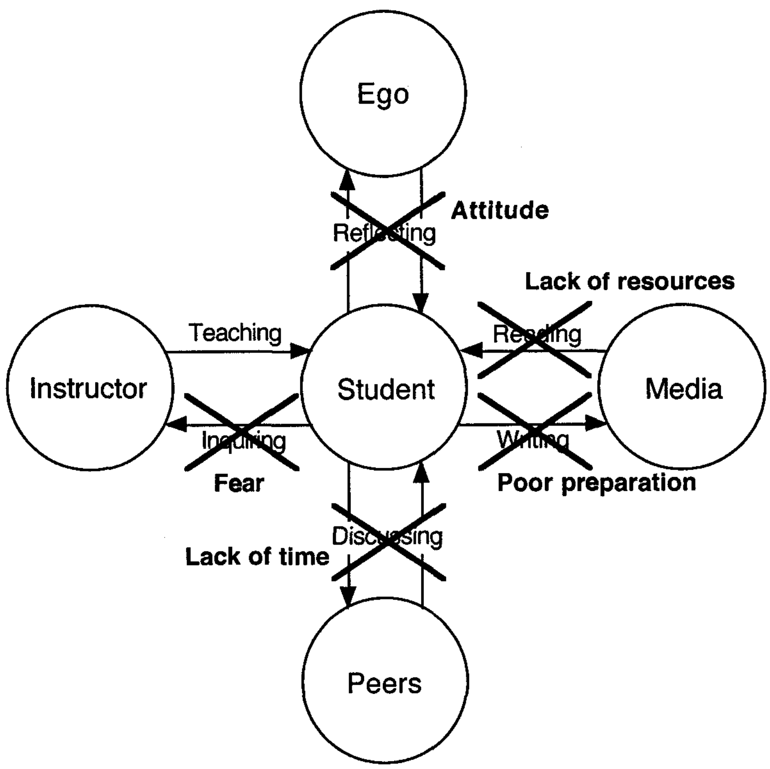

Figure 5. Student-centered factors that may hinder learning.

have poor attitudes towards mathematics and refuse to think about the subject except when absolutely necessary. They may do what they consider to be the absolute minimum amount of work in order to pass the class, which is often much lower than the actual minimum. They may not have money to purchase the textbook, solutions manuals, online access codes, or other ancillary materials. They may be poor readers or writers, or they may have a weak mathematical background. They may be so busy with family and jobs that they have no time to do homework, meet with peers to discuss material, take practice exams, or even attend class. They may feel intimidated by their instructors or fear embarrassing themselves in class by asking questions. They may feel great anxiety before exams or when presenting material publicly. Alas, individual classroom instructors usually can do little to rectify these student-centered problems.

So, what can instructors do to improve the teaching-learning process? Figure 6 illustrates several factors in the instructor's domain of control that may facilitate learning. Recall that pedagogy is the art and science of helping people learn, including the actions,

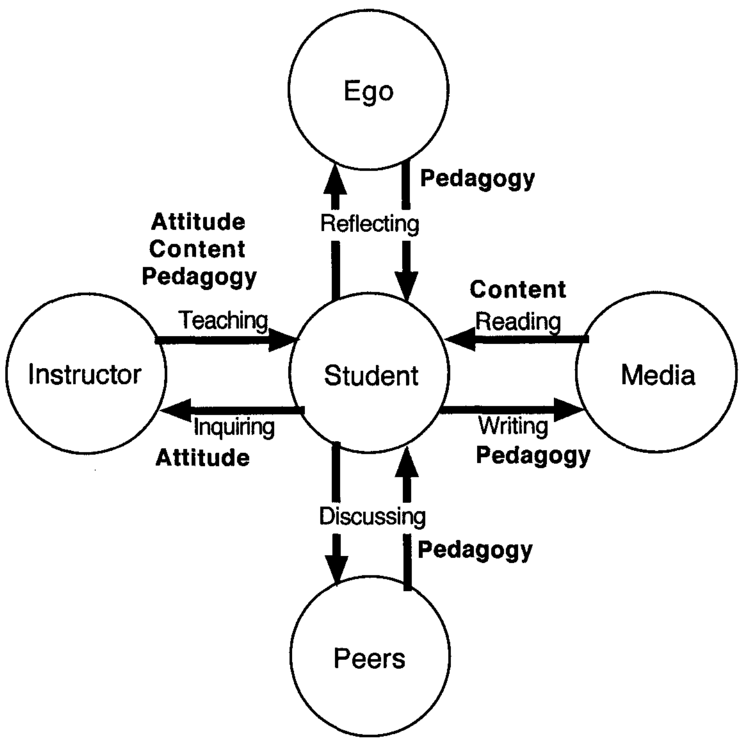

Figure 6. Instructor-centered factors that may facilitate learning.

objects, and attitudes under instructor or institutional control during the teaching-learning process. Perhaps by appropriate manipulation of variables under their control—activities, attitudes, content, etc.—instructors can improve student learning. For example, by including writing and reflection components in their courses, maybe instructors can get students to see the value of self-reflection in their learning. By choosing excellent textbooks or developing their own class notes, they may inspire students to read and interact with the subject in written form. Instructors might form study groups to encourage inter-student discussion and collaboration. Certainly all instructors should exhibit an attitude of helpfulness and interest in student learning. Perhaps certain activities, techniques, or ways of presenting material are more motivating than others for the majority of students.

While the first hypothesis of this study concerns factors external to adult learners that may affect the ability of adults to learn algebra, the second hypothesis deals with a characteristic that is intrinsic to adult learners, namely, the potential for adults to hold the same misconceptions as children when learning algebra. Chapter 2 describes children's algebra misconceptions in detail.

The third hypothesis of this study concerns the external modification of pedagogy based on intrinsic factors in adults. That is, the hypothesis suggests that curriculum should be based in the adult learning characteristics of andragogy, academic fossilization, and metacognition. These characteristics are described in detail in Chapter 2.

It may seem ironic that adult algebra learners experience childlike misconceptions, yet at the same time they assert andragogical characteristics during learning; however, the misconceptions experienced by children during the learning of algebra may not be due to their chronological immaturity, but instead their mathematical immaturity. It is possible that the process of mathematical maturation is similar for most human beings, involving the same stages in roughly the same order, independent of chronological age. This may explain why algebra learners of all ages tend to have similar misconceptions and error patterns.

LIMITATIONS OF THE STUDY

The sample for this study comprises a self-selected set of adult algebra learners at San Diego Mesa College. The study is qualitative and exploratory, and as such it may not be widely generalizable.

CHAPTER 2

REVIEW OF THE LITERATURE

This literature review focuses on three areas of research pertinent to this study: (1) the teaching of algebra, (2) difficulties experienced by children who are learning algebra, and (3) general characteristics of adult learners.

THE TEACHING OF ALGEBRA

This section defines algebra and discusses research on instructional practices.

What Algebra Is

Mathematics is difficult to define. To quote Bertrand Russell (1918, p. 75), "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true." Nevertheless, ambiguity and imprecision, which are rampant in natural language, are anathema to mathematics, so it is not surprising that the natural language term "mathematics" cannot be fully understood naturally; it must be understood mathematically (Smith, 2002). That is, most of mathematics can be understood only to the extent we are willing to venture into its world and use the methods found there. The methods of the physical world cannot provide a true understanding of mathematics.[1]

Just as mathematics evades common definition, the term "algebra" connotes different things for different individuals, from generalized arithmetic for the elementary school teacher to the study of relationships between varying quantities for the middle school teacher to the analysis of abstract structures for the university math professor (Usiskin, 1988). In order to establish common ground for the current study, this study considers algebra to be the mathematical subject that introduces students to variables and manipulations involving variables, solutions of equations, graphs of functions, and a variety of problem-solving techniques.

Algebra has two major components—concepts and procedures—and a knowledge of both is essential for expertise in algebra (Hiebert & Lefevre, 1986). It is typically considered a difficult subject demanding a great deal of practice of techniques; unfortunately, this view may promulgate the fallacy that algebra is a disconnected set of rules devised to deal with different contexts (Tall & Razali, 1993), all of which students must memorize. Teachers hold some responsibility for this fallacy. They often consider the minds of new students to be blank slates

[1] A similar statement could be made about other domains, e.g., music, art, gymnastics, figure skating.

that must be filled with information about their subject. Cognitive science research suggests otherwise. Learning does not occur by depositing information in the mind the way money is deposited in a bank account; rather, it seems to occur through the extension and refinement of conceptual networks or schema that students have previously constructed (Steen, 1989).

Instructional Practices

Teachers' beliefs about students' abilities and learning appear to greatly influence their instructional practices (Thompson, Philipp, Thompson, & Boyd, 1994). Unfortunately, certain discrepancies appear to exist between teachers' beliefs and students' actual abilities. For example, most teachers believe that students have more difficulty with word problems than problems expressed in purely symbolic terms; however, the converse may actually be true. Algebra students often perform more poorly on symbolic problems than on word problems, due to their poor symbol manipulation skills and their use of alternative solution strategies in the word problems (Nathan & Koedinger, 2000).

Not surprisingly, instructional practices can also affect student learning. One study (Henningsen & Stein, 1997; Silver & Stein, 1996) uncovered several instructional factors that had significant influence on the extent to which students were engaged as they carried out mathematical instructional tasks in the classroom. As an example, the study found that the appropriate allotment of time for a task was a predominant influence on engagement level (Doyle, 1986). That is, when enough, but not too much, time was allotted for a task, students became thoroughly engaged in the task without losing interest. On the other hand, when an inappropriate amount of time was allotted for a task, students became frustrated or bored, leading to their cognitive disengagement from the task.

ALGEBRA DIFFICULTIES EXPERIENCED BY CHILDREN

This section discusses research on three types of interdependent difficulties that children tend to experience when they are learning algebra, (1) a poor understanding of the basic components of algebra, (2) a simplistic view of algebraic functions and procedures, and (3) algebraic misconceptions.

Poor Understanding of Basic Components of Algebra

Children may exhibit poor number sense. The National Assessment for Education Progress (O'Sullivan, Reese, & Mazzeo, 1997) found that only 50% of eighth graders were able to correctly identify 35 as the correct solution to $(-7)(-5)$. Trouble in algebra has been connected to a lack of understanding of integers (Moses, Kamii, Swap, & Howard, 1989).

Unfortunately, despite the importance of integers in the middle school curriculum, research indicates that children have great difficulty performing arithmetic operations with them.

Children may have a poor understanding of variables and algebraic conventions (MacGregor & Stacey, 1994). They may not be able to correctly manipulate simple expressions involving variables. For example, two studies (Booth, 1984; Stacey & MacGregor, 1994) found that children habitually simplify $a + b$ as ab. It has also been found that children tend not to use parentheses when writing out complex algebraic expressions, believing that the value of expressions is determined only by the order in which variables and operators are written (Kieran, 1979). Middle school students with a fairly good understanding of variables have been found to assume that different letters must represent different quantities (Kuchemann, 1981). For example, if students are told that rectangle R has side lengths x and y, and then they are asked if R could be a square, it is not unusual for them to say that no, R cannot be a square, because its sides have different lengths.

Children may reject non-numerical answers, assuming that the answer to every problem must be a number. Thus, the symbol $=$ may be perceived as a unidirectional sign preceding a numerical answer[2] (Kieran, 1981; McNeil & Alibali, 2005a; Wagner, 1977). If students are asked to find the perimeter of a polygon with n sides of length two, many believe that they cannot respond until n is given a numerical value. Even when students are willing to accept non-numerical answers, they may expect answers to contain only a single term. For example, if students are told that Team A scored x goals and Team B scored y goals, they may obtain the correct algebraic expression for the total number of goals, $x + y$; however, they may view the answer as improper since it is not a number nor a single term (Booth, 1984, 1988; Chalouh & Herscovics, 1984). Some studies (Matz, 1980; Sfard, 1991) suggest that children insist on numerical or single-term answers due to their experience regarding well-formed answers in arithmetic. An alternate explanation may be that children see operators between terms as commands to perform operations which have not been completed. Many children cannot accept this lack of completion or closure (Collis, 1975; Norton & Cooper, 2001). To be proper, a solution must be of closed form with no unfinished operations; thus, the answer $x + y$ may be considered improper because the addition has not been performed.

Simplistic View of Algebra

Children often have difficulty grasping functions (L. L. Clement, 2001). They may assume that all functions are linear (Markovits, Eylon, & Bruckheimer, 1983), or they may readily dismiss important aspects of functions, such as the domain and range. It is difficult for them to see the equivalence among different representations of functions, e.g., sets of

[2]It is interesting to note that the assignment operator $=$ in computer science is indeed a unidirectional sign preceding an expression that reduces to a specific value.

coördinate pairs, tables of values, graphs, equations, and verbal descriptions (J. Clement, Lochhead, & Monk, 1981; J. Clement, 1982; Lochhead, Eylon, Ikeda, & Kishor, 1985; Markovits, Eylon, & Bruckheimer, 1988; Mestre, Gerace, & Lochhead, 1982; Mestre & Lochhead, 1983; Mestre & Gerace, 1986; Rosnick & Clement, 1980; Rosnick, 1981; Soloway, Lochhead, & Clement, 1982; Teachey, 2003).

Children may go to great lengths to avoid using algebraic methods to solve algebra problems. Given an equation to solve, they often prefer to use informal methods such as inspection or trial-and-error substitution (Amerom, 2003; Booth, 1981; Ekenstam & Nilsson, 1979; Ginsburg, 1975; Kieran, 1988; Petitto, 1979). For instance, rather than subtracting 3 from both sides of $3 + x = 7$, middle school students may focus on finding a number x such that $3 + x$ is equal to 7. While this procedure is legitimate, it is of limited use for even slightly more sophisticated equations, e.g., $3 + x = 8 - x$.

Algebraic Misconceptions

Elementary algebra instructors and researchers observing children semester after semester conclude that each cohort of students tends to have the same misconceptions (Tall & Thomas, 1991) and make the same types of errors (Lerch, 2002). Algebra errors seem to be independent of geographical location. For example, students in Fiji, Israel, and Japan experience similar error patterns as students in the United States and other countries (Lochhead et al., 1985; Mestre et al., 1982; Tall et al., 2000). Such errors include the following.

- The conflict of natural language and the symbolism of algebra, in which algebraic expressions such as $3x + 2$ and $2 + 3x$ are read from left to right, but the former is evaluated from left to right while the latter is evaluated from right to left. Some students will insist on evaluating the latter from left to right, producing the incorrect response $2 + 3x = 5x$.
- Another facet of the natural language-algebraic symbolism conflict, in which algebraic expressions such as ab are read aloud as "a and b" and then promptly rewritten incorrectly as $a + b$.
- The name-process dilemma, in which the equivalent expressions $3(a + b)$ and $3a + 3b$ are not identified, because they involve different sequences of operations, i.e., an add followed by a multiply for the former expression, and two multiplies followed by an add for the latter.
- Academic fossilization, in which students incorrectly recall mathematical procedures. For instance, some students taking algebra for the second time treat the expression $(2x + 3)^2$ as $(2x + 3)2$, obtaining the incorrect expression $(2x + 3)^2 = 4x + 6$.

Misconceptions are incomplete half-truths that students use to make sense of the world (Mestre, 1987). They are repeatable, explicit, incorrect features of student knowledge

(Leinhardt, Zaslavsky, & Stein, 1990) that seem to be particularly devastating for algebra learners, since learners hesitate to discard them and replace them with correct procedures and knowledge (Resnick, 1982). This supports what Ausubel noted nearly 40 years ago, that correct prior learning may be the most important factor in future learning (Ausubel, 1968). Three misconceptions frequently held by children algebra learners are the following.

- The *expression as procedure* misconception.
- The *letter as label* misconception.
- The *graph as path* misconception.

These are explained below.

The *expression as procedure* misconception prevents children from seeing algebraic expressions as objects. To them, algebraic expressions are procedures or directions for performing computations (Tall & Razali, 1993). They see the symbol $+$ as a command to perform an operation, and the symbol $=$ as a command to write down the answer (Behr, Erlwanger, & Nichols, 1980; Carraher, Schliemann, & Brizuela, 2000; Ginsburg, 1977; McNeil & Alibali, 2005b). Also known as the name-process dilemma (Davis, 1975), this misconception impedes students from performing high-level algebraic manipulations of large expressions. Some studies (Alibali, 1999; McNeil & Alibali, 2000) estimate that as many as 75% of children have the *expression as procedure* misconception.

When children consistently misinterpret letters as labels or units instead of variables, they suffer from the *letter as label* misconception. An arithmetic teacher may write "25 m" to represent 25 meters; however, the almost identical expression $25m$ means something quite different in algebra. Many students—even those who have already passed algebra—display confusion in the use of letters (Booth, 1988). This confusion often reveals itself when students are asked to convert a sentence into a mathematical expression (Lochhead & Mestre, 1988), and it appears to carry on long after public school years. Indeed, when college engineering students were asked to identify an equation corresponding to the statement, "There are six times as many students as professors," about a quarter of them chose the incorrect answer, $6S = P$ (J. Clement et al., 1981; Rosnick, 1981). Among school children, it appears that at least two-thirds have the *letter as label* misconception (Mestre & Gerace, 1986).

Children who have the *graph as path* misconception may favor graphs that exhibit the appearance of the physical situation represented by the function, rather than the appropriate graphical relationship between dependent and independent variables. For example, when children were asked to sketch the graph of speed versus time for a bicycle ascending and descending a steep hill, they tended to sketch a picture of a hill (Bowers & Doerr, 2001; Dugdale, 1993; Lapp & Cyrus, 2000). This misconception, also known as the iconic interpretation misconception, is held by more than half of school children (Leinhardt et al., 1990).

Characteristics of Adult Learners

This section describes three categories of adult learning characteristics: andragogy, academic fossilization, and metacognition.

Andragogy

Malcolm Knowles, a vanguard of adult learning research, called his academic field of study andragogy. The term comes from the Greek for "adult-leading," just as pedagogy comes from the Greek for "child-leading." Andragogy today is usually described as the art and science of helping adults learn (Merriam, 2001; Rachal, 2002; Zmeyov, 1998). In his theory of andragogy, Knowles proposed that adult learners share the following characteristics (Knowles, 1980).

- Self-concept
- Experience
- Readiness to learn
- Time
- Orientation to learning

These are described below.

Self-Concept

Adults need to be responsible for their own decisions and to be treated as capable of self-direction. According to Knowles, a person's self-concept moves from a state of dependency to a state of independence as the person matures. The point at which an individual becomes essentially self directed is the point at which that person psychologically becomes an adult (Knowles, 1984). Thus, being self directed is a quality of being an adult, and removal of the opportunity for self-direction during instruction may result in learning-inhibiting tension. On the other hand, showing adults how to direct themselves through information may inspire and energize their learning.

Experience

Adults need to learn experientially, including from mistakes. Adults have accumulated large collections of experiences that represent a rich resource for learning; indeed, such experiences are of great worth to most adults. Nevertheless, some of these experiences may contain bias or misconceptions that may interfere with learning. Relating topics to the experiences of learners may facilitate learning for adults.

Readiness to Learn

Adults are ready to learn what they need to know to deal effectively with their social roles. While children feel strong peer pressure to learn specific subjects at specific times because all the other kids are doing the same thing, adults do not experience this same social pressure. Instead, they tend to feel ready to learn when their roles in society demand it. Adults need to know why they must learn something before they are ready to learn it.

Time

Adults learn best when the topic is of immediate value to them. Children are expected to learn things for application at a future date ("You'll need this later"). Adults do not have as much patience for postponed application of learning. They expect what they learn to have immediate application; otherwise, why bother learning it?

Orientation to Learning

Adults approach learning as problem solving rather than passive acquisition of abstract concepts. Adults are motivated to learn in order to solve problems or perform tasks in their lives. They are less likely to care about a subject's abstract concepts that will probably be of no practical use to them.

Academic Fossilization

The theory of andragogy presumes that adults learn from experience and that they use their collection of experiences as a learning resource; however, it is possible that such experience is not pristine. Bias, false presupposition, faulty recollection, and missing information can affect how well experience aids learning. When learning has not taken place completely so that the learner has only a partial truth, the learner has a misconception. When a learner exhibits erroneous mathematical behaviors after forgetting previously learned knowledge or the conditions under which it applies, the learner suffers from academic fossilization (Lerch, 2002).

Vygotsky (1978) introduced the general concept of fossilization to describe the solidification of behaviors in an individual after the individual has forgotten the situations in which the behaviors were learned. Academic fossilization, then, is simply Vygotsky's fossilization restricted to academic situations. Students with academic fossilization are left with only vestiges of disconnected skills and concepts, which they may apply incorrectly (Karsenty, 2002). Academic fossilization may be especially pervasive in college remedial algebra courses, since students in such courses have invariably studied the material years before. Unfortunately, when they recognize concepts being introduced during instruction, they may assume that they already know the material and thus not pay careful attention as it is presented.

Metacognition

The ability of individuals to be aware of and regulate their own knowledge and learning is called metacognition (Borkowski, Carr, Rellinger, & Pressley, 1990; Flavell, 1979; Paris & Winograd, 1990; Pressley, Borkowski, & Schneider, 1987). Research indicates that as people mature, they tend to become more aware of their own thinking processes (Duell, 1986); however, Schoenfeld argues that metacognitive skills should be explicitly taught to adult learners (Schoenfeld, 1987), since metacognition plays such a critical role in successful learning. Thus, not only should students acquire mathematical knowledge, they should also learn how, when, and where to apply such knowledge. To accomplish this, instructors should probably present explicit instruction in the skills of metacognition.

Schoenfeld outlined four skill categories that are essential for success in mathematics (Schoenfeld, 1985).

- Resources: A student must have propositional and procedural knowledge of mathematics.
- Heuristics: A student must use strategies and techniques for problem solving, such as working backwards or drawing figures.
- Control: A student must be able to make decisions about when and what resources and strategies to use.
- Beliefs: A student must have an appropriate world view in order to determine the approach to solving a problem.

Traditionally, math instructors have felt a duty to impart resources and some heuristics to their students. Students were expected to develop further heuristics, as well as appropriate control and beliefs, through experience in working with mathematics and the process of mathematical maturation. Adult learners, however, may not have the time required for the natural development of heuristics, control, and beliefs. It may be more suitable for instructors to provide explicit training in these areas to adult learners.

The present study concerns adult algebra learners' difficulties. Unfortunately, not much research nor theory exists on this topic. Thus, the theoretical framework for this study has drawn on theories and bodies of research that are somewhat related, but not perfectly matched, to the topic. These include research on child algebra learners' difficulties, especially child algebra misconceptions, and general adult learning theories, specifically andragogy, metacognition, and academic fossilization. Furthermore, this study has assumed the existence of pedagogical and other factors that influence the learning of algebra by adults.

CHAPTER 3

METHODOLOGY

This study incorporates a hypothesis-guided, grounded theory-based methodology in which adult algebra learners were interviewed and observed doing algebra. Grounded theory is an established qualitative research strategy that is particularly well suited for research on learning because it offers a functional means of constructing theory grounded in the day-to-day reality that educators face (Babchuk, 1997). As in most grounded theory studies, a final product of this study is a substantive theory grounded in the data obtained.

DESIGN OF THE INVESTIGATION

The research design of the proposed study draws on grounded theory methodology (Glaser & Strauss, 1967), especially the more structured version espoused by Strauss and Corbin (1994) in which a theoretical framework is permitted. In the original incarnation, still promoted by Glaser, no precategorization or pre-theorization was permitted. Theory was required to flow directly from data. The later version of Strauss and Corbin allows the forming of hypotheses to initiate category formation, although theory still emerges substantially from data.

Grounded Theory

The focus of this study is to generate a theory of adult algebra learning from data collected in the field, informed—but not limited—by previously developed theory on algebra learning among children and general adult learning. The grounded theory methodology is effective for disciplines in which complex phenomena are to be studied without being manipulated to suit a predetermined research design, especially if there exists no prior theory to explain the phenomena. Adult education is such a discipline, characterized by a strong commitment to practical techniques and the lack of a well-developed theoretical foundation. Shulman (1984) argues that education is not even a discipline but a field of study comprised of phenomena, events, institutions, problems, persons, and processes, that allows, if not demands, inquiries of many kinds.

The rationale behind the grounded theory method is that theory should be grounded in empirical evidence; that is, it should evolve from data, rather than be developed *a priori* and then tested against data. Grounded theory derives its theoretical bases from the philosophical movements of pragmatism and interactionism (Blumer, 1969), in which truth is seen as emerging from interactions among human pragmatic actors, rather than some immutable, underlying object that governs actions. Pragmatists and interactionists do not see truth as an objective reality that is viewed the same by every unbiased person; instead, truth can only be interpreted, and never without bias.

Grounded theory is thus a qualitative method of inquiry concerned with understanding action from the perspective of the human actor. Its procedures are not statistical, but interpretive, beginning with the assumption that researchers can access reality only through social constructions such as language, consciousness, and shared meanings. Grounded theory research differs from traditional scientific method-based research that begins with a theory, i.e., a model based on a set of assumptions, and uses data obtained from the manipulation of variables to support or refute the theory (see Figure 7). The goal of grounded theory is to

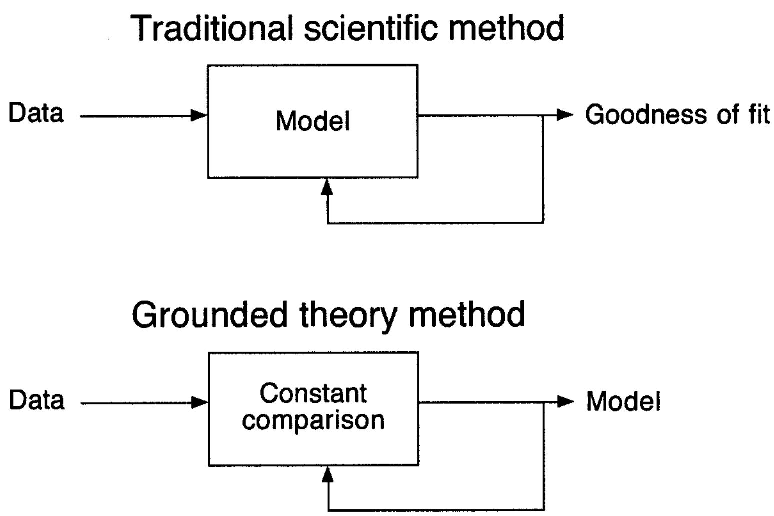

Figure 7. Scientific method versus grounded theory.

generate a theory grounded in the substantive data collected during the study; that is, the approach begins with data and produces a theory *sans* variable manipulation. Grounded theory also differs from other qualitative research approaches such as biography, phenomenology, ethnography, and case study, in the way data are collected, the type of sample used, and the method employed to analyze data (Creswell, 1998). In particular, grounded theory is the main qualitative research tradition that utilizes the constant comparison method.

Constant Comparison

An essential feature of grounded theory research is the constant comparison method, a procedure for analyzing data by comparing new data to previous data in order to form categories. The constant comparison method is an iterative process usually described as consisting of four stages (Lincoln & Guba, 1985)

1. Compare concepts with categories
2. Integrate categories
3. Delimit the theory
4. Write the theory

as shown in Figure 8. The feedback loops in the figure illustrate common iteration paths of the process; however, the process is flexible enough to allow feedback from the output of any

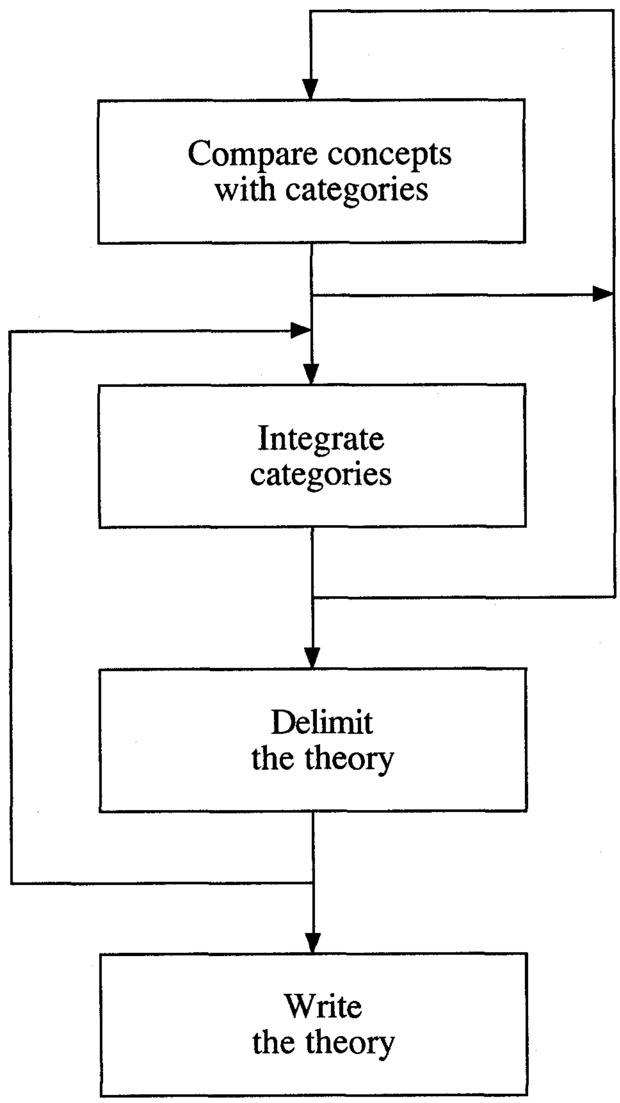

Figure 8. Flow diagram of the constant comparison method.

stage to the input of any prior stage. The four stages of constant comparison are discussed below.

Compare Concepts & Categories

In the first stage of the constant comparison method, the investigator transcribes recorded data and then analyzes the transcribed data through open coding (Strauss, 1987). In this process, the stream of transcribed data is chopped up, or tokenized, into small textual bits called tokens. Each token contains one or more concepts of interest to the study. The investigator creates categories by clustering these tokens following whatever criteria seem appropriate. Since the final theory provided by the study emerges from categories, data categorization is a crucial step in the data analysis process. Several sources inform category formation, including inferences from the data, research hypotheses or emergent research questions, theoretical issues, previous knowledge, intuition, and imagination (Dey, 1993).

The discovery of patterns in the data is a creative process that requires careful judgment about what is significant in the data (Patton, 2001). The correspondence between data and categories undergoes continuous refinement, and criteria for including or excluding data concepts become more precise as the analysis progresses. The refinement of categories requires that multiple perspectives be considered during the research inquiry. As categories form, the investigator may hone the selection of potential participants through theoretical sampling, choosing individuals who will test the limits of the categories, i.e., the conditions under which categories are valid.

Integrate Categories

In the second stage of constant comparison, the investigator begins axial coding of the data. This is done by reassembling the categorized data in new ways to explore relationships between categories and properties. The investigator may use memos to record any connections identified between categories. Theoretical sampling may be employed to select participants who can aid setting boundaries for the categories by exposing conditions under which the emerging model holds or does not hold. The goal of this stage is to discover a category that appears central to the study. This is the core category.

Delimit the Theory

The third stage of constant comparison begins when the collection and analysis of additional data adds little or no new information about categories. Called category saturation, this situation typically occurs after 20 to 30 interviews for grounded theory studies (Creswell, 1998). At this point, the investigator moves on to selective coding, in which he attempts to identify the story line of the study by relating all categories to the core category and elucidating the historical, social, and economic influences on the central phenomenon of the study associated with the core category.

Write the Theory

The final stage of the constant comparison method provides the primary product of the study, i.e., a substantive theory grounded in the data of the study. During data collection and category formation and evolution, the investigator may have kept field notes and memos on category properties and connections. In conjunction with the story line just identified, these written documents help form the basis of the theory. All that remains for the investigator is to weave these materials together into a coherent argument.

In summary, grounded theory research assumes that events do not occur randomly, and a careful analysis can reveal explanatory theories that coincide with observed events and predict future events. Coding reveals categories, and memoing adds relationships that link categories to each other (Dick, 2002). Theory emerges from these relationships. As in the world of Tlön from the epigraph, grounded theory does not view truth as being comprised of specific, concrete objects that proffer the same understanding to every individual; rather, truth is interpreted distinctly by each individual from a heterogeneous series of independent acts.

Participants

Since this study's aim was to uncover difficulties faced by adult algebra learners, participants for the study were adults engaged in algebra learning. The vast majority of adults who learn algebra in the U.S. do so at two-year colleges. Across the nation, about 10% of two-year-college students are enrolled in Beginning or Intermediate Algebra (Lutzer et al., 2006). At San Diego Mesa College, approximately 50 sections of Beginning Algebra (Math 95) and Intermediate Algebra (Math 96) serve over 1500 students every semester out of a total enrollment of 20,000 (SDCCD IRP, 2006), or 7.5%, slightly below the national average.

In the summer and fall of 2005, I recruited participants from classrooms at San Diego Mesa College in which Math 95 and Math 96 were being offered. At the start of each term, I emailed all Math 95 and Math 96 instructors at Mesa College requesting that they allow me five minutes at the start of class to announce my study and vocally recruit participants. During the five minutes in each class, I read a script (see Appendix A) announcing the study, then I distributed a recruitment flyer (see Appendix B) describing how students could contact me later if they were interested in participating. I also posted recruitment flyers in Math 95 and Math 96 classrooms on campus.

As noted on the recruitment flyer, the criteria for inclusion as a participant in this study were the following.

1. Participant is at least 18 years old.

2. Participant is enrolled in Math 95 or Math 96 at San Diego Mesa College during the summer or fall of 2005.

To avoid arbitrary restrictions on the sample for this exploratory study, class performance was not a criterion.

I received responses from 29 individuals. I interviewed all 29, and the first 25 formed the sample. The data from the last four interviewees were not used in the study due to prior achievement of category saturation. The sample included 12 females, 13 males, 2 Asians, 5 Blacks or African-Americans, 3 Latinos or Hispanics, and 15 Whites. The age of participants ranged from 18 to 48, with a mean of 27.8. The mean age of Mesa College students during spring 2006 was approximately 26.8. The relative frequencies of gender, ethnicity, and age for the sample and for the population of spring 2006 Mesa College students are summarized in Table 1. Participation in this study was completely voluntary, and identities of the participants

Table 1. Characteristics of the Sample and of Spring 2006 Mesa College Students

Gender	Sample %	Mesa College %
Female	48	55
Male	52	45
Ethnicity	**Sample %**	**Mesa College %**
Asian	8	21
Black	20	6
Latino	12	17
Native American	0	1
White	60	42
Unknown	0	13
Age	**Sample %**	**Mesa College %**
Under 18	0	2
18–19	16	18
20–29	56	58
30–39	16	13
40–49	12	6
Over 49	0	3

were kept anonymous. Although the sample was self selected, it is fairly representative of the San Diego Mesa College student population in terms of gender and age. Due to the ethnicity category "Unknown," it is difficult to determine how well the sample's ethnicity frequencies match those of the Mesa College student population; however, all major ethnicity categories were well represented in the sample.

Data Collection

Investigators cannot observe directly how knowledge is represented in the mind. They are limited to making inferences based on external observations. Assuming that mathematical knowledge is represented mentally in a structured manner (Gardner, 1985), e.g., via the schema construction idea discussed on page 11, it may be possible to determine the degree of understanding of a mathematical concept or procedure by probing the mental structure repeatedly through inquiries. The subject and depth of successive inquiries are determined by responses to previous inquiries, hence a mechanism for observing responses and adjusting tacks of inquiry is required. Rigid techniques such as tests, questionnaires, and fully structured interviews do not provide this mechanism; therefore, they may not be the best choice for getting at the degree of understanding of a concept or procedure. On the other hand, a semistructured, real-time technique such as the clinical interview (Ginsburg, 1997) is flexible enough to provide the necessary inquiry adjustment mechanism for exploring understanding.

In this study, I gathered data from participants through audiotaped clinical interviews. Appendix C contains institutional review board permissions for this study from San Diego State University, the University of San Diego, and San Diego Mesa College. The clinical interview is an open-ended technique that allows just-in-time mini-hypothesis testing so the investigator can focus on and deeply investigate phenomena of interest. During clinical interviews, participants act as informants, responding to interview questions designed to identify strategies the participant uses when performing tasks (McLoughlin & Lewis, 1994). The clinical interview technique is conducted for the benefit of the investigator rather than the participant, so the investigator generally does not assist the participant in responding but merely observes responses.

When potential participants contacted me by email or telephone as indicated on the recruitment flyer, I read them the informed consent statement (see Appendix D), then we arranged a date and time for a one-hour clinical interview in my Mesa College office, H209B. All interviews were scheduled outside of participants' class hours. I interviewed up to nine participants per week during the summer and fall of 2005. I determined that the categories that emerged became saturated after 25 interviews.

During each interview, the participant and I sat next to each other at my desk, which held some sheets of paper, a pen and a pencil, and a small audio recorder consisting of a Griffin Technology iTalk microphone connected to an Apple iPod MP3 player. The iPod's capacity was 40 gigabytes, more than sufficient for 25 hour-long interviews requiring approximately 100 megabytes each. I provided the participant a copy of the informed consent statement and verified that the participant understood the entire document, especially the fact that participation was completely voluntary and anonymous and that the participant could discontinue participation at any time for any reason without penalty. In order to demonstrate sensitivity towards all participants, especially those who were ethnically or culturally different from myself, I assured them that:

- I respect and value ethnic, cultural, and other differences among people.
- I reject cultural and ethnic stereotypes.
- If they perceive disregard for their culture or ethnicity or feel discrimination during the study, they may express their concerns or discontinue their participation at any time without penalty.
- The ultimate goal of this study is to improve algebra instruction for all adult learners at Mesa College, regardless of ethnicity or culture, and their participation will aid in this effort. The study does not focus on any particular gender, ethnicity, culture, or individual. The only reason that ethnicity data are collected is to verify that the sample is diverse.
- I will not judge their performance or opinions in any way. I am only interested in uncovering conceptions and views held by adult algebra learners.

At this point in the interview, I turned on the audio recorder and asked the participant's age and ethnicity. I assured participants that their responses would be completely anonymous, that they did not need to answer any question they did not like, and that their grade in Math 95 or Math 96 would not be affected in any way by their participation or lack of participation in this study. I then proceeded as outlined in an interview guide (see Appendix E), requesting the participant's thoughts and opinions on algebra instruction and learning, giving the participant algebra problems to solve on paper, and asking the participant to describe thinking that occurred during problem-solving. Although the primary goal of the clinical interviews was to get participants to talk as much as possible about whatever they wanted regarding the difficulty of learning algebra, the three parts of the interview were designed to elicit information from participants regarding the three research hypotheses.

Part 1 of the Interview

During the first 15 minutes of the interview, I asked participants to describe what conceptual and affective difficulties they face when learning algebra. I elicited participant responses by prompting with questions such as, "What is the most difficult thing about learning algebra?" or "What are the top three things you like or dislike about algebra?" This was a chance for participants to vent their frustrations, describe their fears, complain about or praise their instructors, or express their joy or distaste for mathematics. Information on influential pedagogical factors gleaned from this part of the interview addressed the first hypothesis of the study.

Part 2 of the Interview

The next 25 minutes of the interview were devoted to solving three algebra problems on paper. Each problem was printed at the top of its own sheet of paper, and participants were given one problem to work on at a time. After reading the statement of a problem to a participant and requesting the participant to voice his or her thoughts while working the problem, I remained silent as the participant worked. Often participants would ask for assurance that what they were doing was correct, and I would tell them that I was not there to assist them, but only to observe their actions. When participants became frustrated or wanted to give up on a problem, I would ask them to describe what they were feeling and why they felt the way they did. Once it was clear that a participant could make no more progress on a problem, I would put away their work and give them a new sheet containing the next problem.

This part of the interview examined the second hypothesis and part of the third, namely, the extent of participants' algebraic misconceptions, their extent of academic fossilization, and their metacognitive skill. The problems were derived from the literature as prototypical problems for each of the three misconceptions included in the second hypothesis.

PROBLEM 1

The first problem was designed to explore the extent to which adults possess the *expression as procedure* misconception (Tall & Razali, 1993):

> On paper, solve $1 + (x + yz) = 2(x + yz)$ for x. Explain your thinking.

Participants who see the expression in parentheses as a single object could then subtract it from each side, leaving the easily solved linear expression $1 = x + yz$. On the other hand, participants who felt compelled to perform the operations indicated by plus signs or implied multiplication would probably not be able to treat the expression in parenthesis as a single object. The expression on the left side, although similar in form to the distribution of multiplication over addition on the right side, is not an example of distribution. Participants with academic fossilization related to distribution might err by performing an incorrect distribution on the left side. The statement "Explain your thinking" was included in each problem in an attempt to observe the participants' metacognitive activity during the solution of the problems.

PROBLEM 2

The second problem was selected to explore the extent to which adults possess the *letter as label* misconception (J. Clement et al., 1981):

> Suppose there are six times as many students as professors at Mesa College. Write this relationship mathematically using variables S and P. Explain your thinking.

I explained verbally to each participant that the variables S and P should represent the number of students and the number of professors, respectively. The most common erroneous answer to this well-known problem is $6S = P$; however, participants with a firm understanding of variables should recognize that S and P are not labels, hence "$6S = 1P$" does not mean "six students to one professor." Participants with good number sense would realize that S must be larger than P because there are more students than professors, hence "$6S = P$" could not possibly be correct.

PROBLEM 3

The third problem was selected to explore the extent to which adults possess the *graph as path* misconception (Dugdale, 1993):

> Sketch a graph of speed (y) versus time (x) for a bicycle going up a steep hill and down the other side. Explain your thinking.

I advised all participants that they could imagine any type of hill they desired, but that they should describe their hill verbally. I also reminded them that the y-axis was to be the vertical axis and the x-axis was to be the horizontal axis since the problem required a graph of y versus x. Participants with the *graph as path* misconception would probably draw a picture of a hill for this problem, while those with strong graphing skills would typically produce some sort of an increasing function.

Part 3 of the Interview

The final 20 minutes of the interview were dedicated to exploring part of the third hypothesis, i.e., the extent to which participants exhibited the following andragogical traits during the learning of algebra: (1) self-directedness, (2) the need to learn from experience, (3) readiness to learn, (4) the need for learned material to have immediate application, and (5) preference to learn through solving practical problems over acquiring abstract concepts. To get at this information, I asked participants questions such as the following.

1. Would you like to be involved in determining how the course is taught and how your grade is determined?
2. How does your past experience affect your learning?
3. Is it important that what you are learning has personal relevance to you?
4. Should learning concentrate on solving problems or gaining knowledge of abstract ideas?

It was expected that the first question would lead to a discussion about how much self-direction and control the participants wanted in the course and how much responsibility they expected their instructors to give them. The second question was designed to bring up a discussion of learning from mistakes and the utility of past experience during current learning. Presented with the third question, it was hoped that participants would reveal their readiness to learn as well as the extent to which they required learned material to have immediate application. The fourth question was devised to elicit information about how participants liked to learn, i.e., solving practical problems or acquiring abstract concepts.

Interviews proceeded in an informal conversational style, with an element of flexibility built in to allow me to explore serendipitous avenues of inquiry that could appear during the interviews. Per the clinical interview technique, I attempted to confirm or refute emerging ideas via mini-hypothesis tests *in situ*, asking discriminating questions or requesting that participants solve specific problems and describe their thinking.

To conclude the interviews, I turned off the audio recorder, gave the participants $10.00 in cash, had them initial a receipt for the payment, and thanked them for assisting with the study. I also debriefed interested participants about the problems they solved. If, during the course of the interview, I learned that a participant might have a learning disability, I provided the participant with a handout of information regarding tutoring services available on campus, as well as contact information for Mesa College Disabled Students Programs and Services, the campus organization that provides assistance to students with learning disabilities (see Appendix F).

Data Analysis

The flow diagram of Figure 9 was constructed to illustrate the data collection and analysis process for this study. Data collected during interviews included participants' opinions and thoughts on algebra instruction and learning, self-reflections on algebra learning difficulties, thoughts regarding the types of algebra misconceptions held by children, thoughts regarding adult learning characteristics, algebra problem-solving performance, and descriptions of thought processes during problem solving. Data consisted of digitally recorded audio and problem solutions written by participants.

To perform open coding, I parsed the data from each interview into *in vivo* tokens comprised of verbatim phrases uttered by participants containing a nugget of information related to the learning of algebra. Each participant yielded between 20 and 109 tokens. I then sorted the tokens according to content, forming major categories to house all the tokens. Upon completion of each interview, I incorporated the tokens and their categories into the corpus of previously collected data through the constant comparison method, adding new categories as needed and periodically combining categories when their equivalence became clear. As the corpus of interview data grew, I performed axial coding, in which I searched for connections between the categories. Once interviews appeared to add little to category formation, i.e., category saturation was achieved, I terminated the interview phase and identified the core

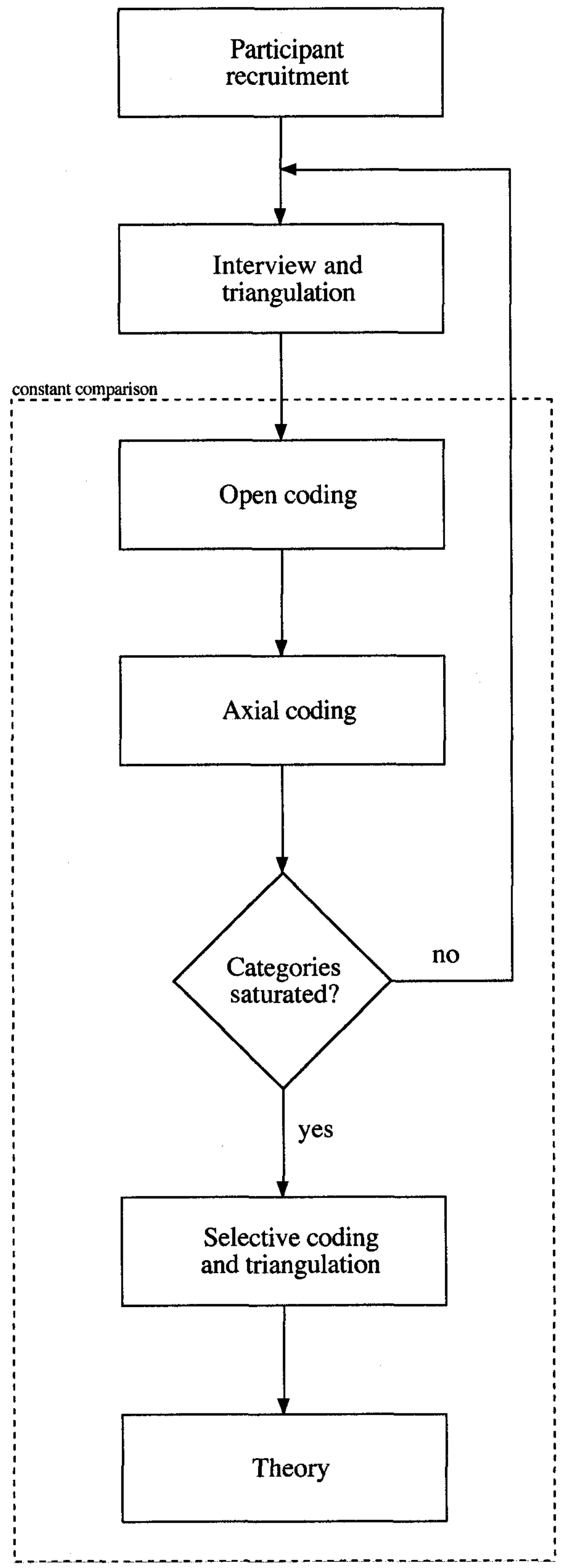

Figure 9. Flow diagram of data collection and analysis activities.

category. I then began selective coding of the data, attempting to relate the core category to all other categories. From this integration of categories, I identified the story line, which is the principal product of the study, a theory grounded in substantive data.

An essential aspect of my data collection and analysis activities was the process of triangulation. In order to verify that my analyses were reasonable, I discussed emerging concepts and categories with later participants during the final portion of their interviews. I did this in the latter portion of the interviews to avoid compromising participants' responses.

A second source of triangulation was the administration of an anonymous written survey in two Math 95 and two Math 96 classrooms. The survey contained 85 Likert-scale items, where each item was a category derived from open coding. The 24 students taking part in the survey were asked to determine whether they strongly agreed, agreed, disagreed, or strongly disagreed with each item as it pertained to themselves personally.

CHAPTER 4

RESULTS AND DISCUSSION

This chapter contains a presentation and discussion of the analysis process and results. Findings are discussed in general, as well as in conjunction with the research hypotheses of the study and with demographics of the sample.

PRESENTATION OF THE FINDINGS

The goal of this section is to describe and discuss the data analysis elements emanating from application of the constant comparison method, i.e., the tokens, categories, supercategories, and core category obtained from the data. The section also presents and discusses the results of the triangulation survey.

Open Coding

Figure 10 illustrates the process of open coding carried out for this study. The tiny text

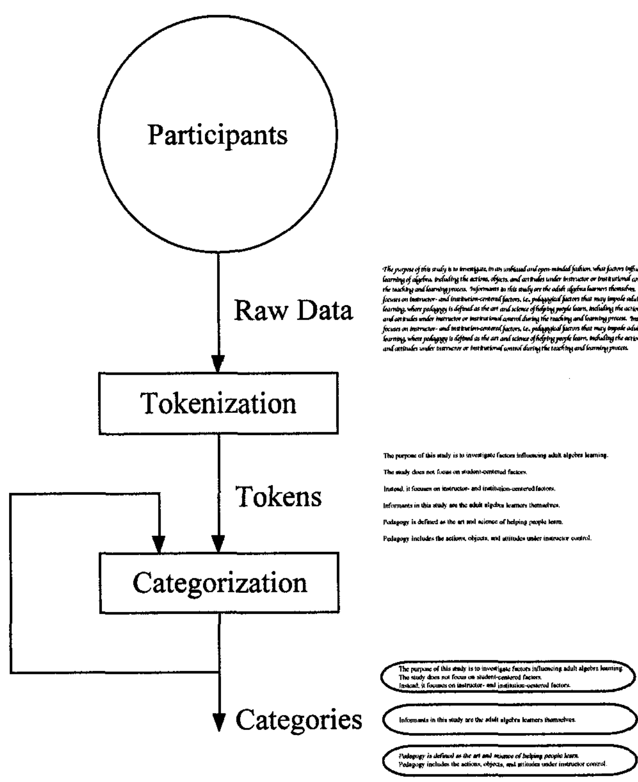

Figure 10. Open coding transforms raw data into categories. (The tiny text is not meant to be read.)

to the right of the flow diagram is not meant to be read. It is there to provide a graphical representation of how data might appear at each level of analysis.

Through clinical interviews, I collected raw data consisting of participants' spontaneous spoken statements and responses to prompts. I separated the continuous stream of spoken data into verbatim phrases uttered by each participant that seemed particularly relevant to difficulties in algebra learning. These *in vivo* phrases became the tokens of the analysis. Examples of what raw data, tokens, and categories might look like are shown in miniature to the right of the flow diagram in Figure 10.

The number of tokens obtained from each participant varied from 20 to 109, with a median of 43, as illustrated in Figure 11. The cumulative number of tokens obtained from

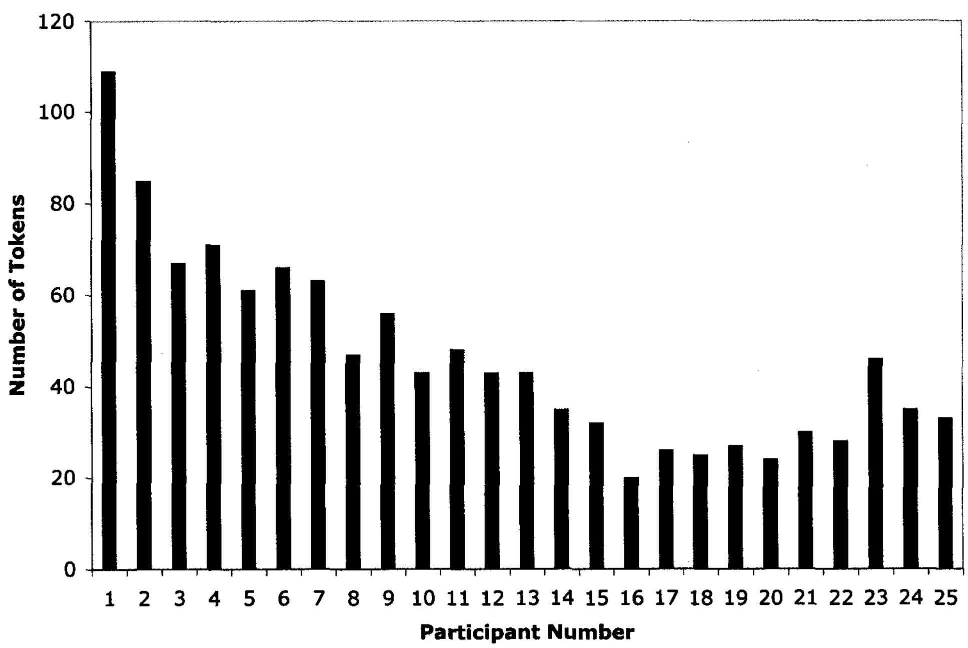

Figure 11. The number of tokens obtained from each participant.

participants is shown in Figure 12.

After tokenizing the raw data, I compared tokens with categories already obtained from the analysis of previous participants' data, and upon finding semantic agreement, I aggregated the new tokens to the old categories. Whenever tokens from the new participant could not be matched to extant categories, I created new categories for those tokens. Once all tokens had been categorized for a participant, I examined the set of categories, merging similar categories when possible. Figure 13 illustrates the number of categories in existence after this merging process was carried out for each participant. Note that the number of categories rose to a maximum of 179 halfway through the set of data, and then it fell, leveling

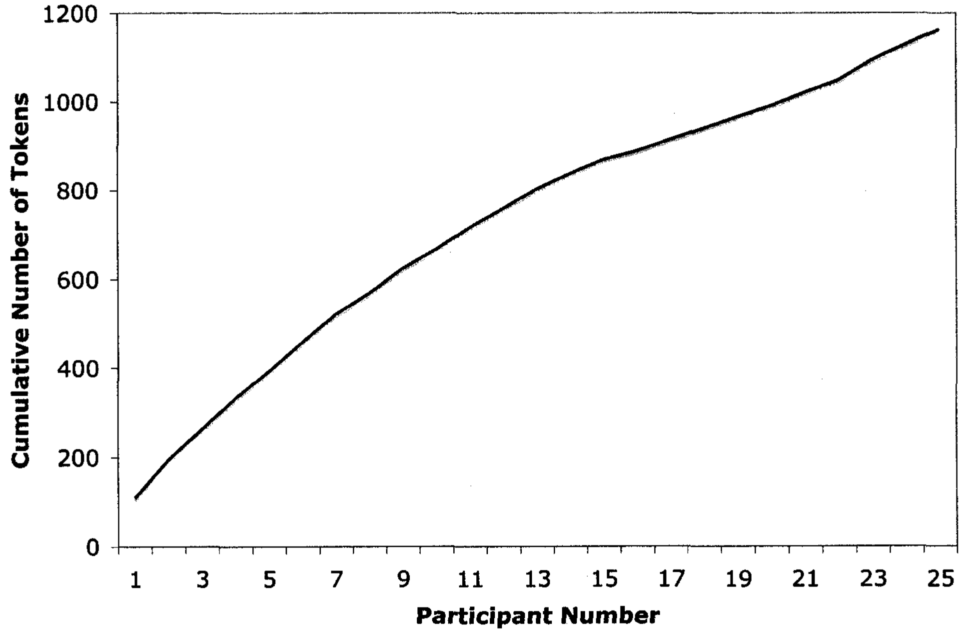

Figure 12. The cumulative number of tokens obtained after each participant's data was analyzed.

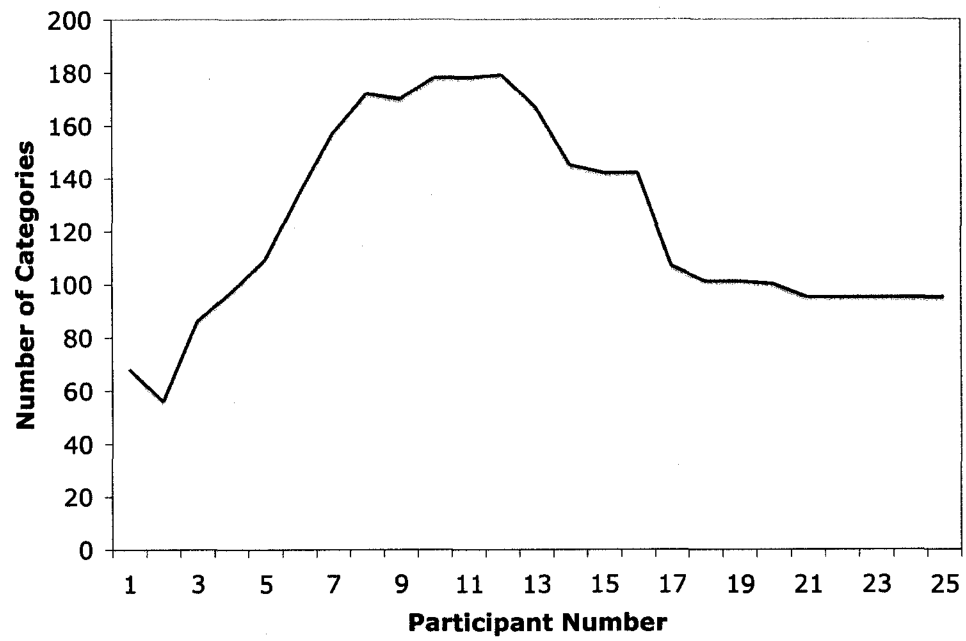

Figure 13. The number of extant categories after each participant's data was analyzed.

off at 95 over the final 20% of the data. This leveling off in the number of categories is an indicator that saturation was achieved.

We say that a participant *supported* a category if he or she generated at least one token belonging to the category (we also say that the participant generated the category). The more tokens a participant generated for a category, the higher the level of support the participant gave the category. The number of final categories that each participant supported varied from 18 to 40, as shown in Figure 14. The number of final categories supported by participants

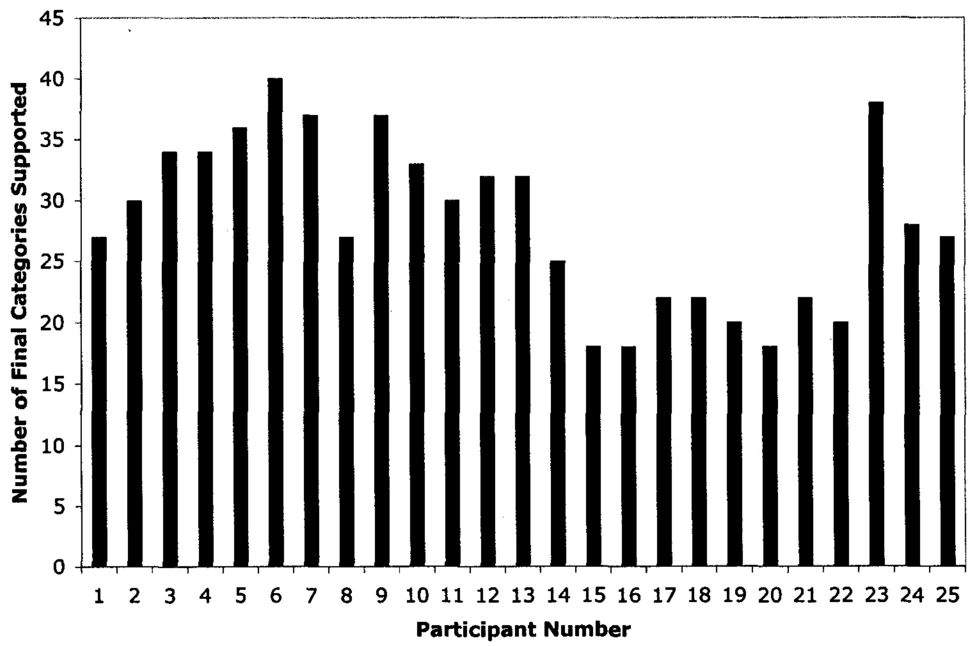

Figure 14. The number of final categories supported by each participant.

grew monotonically from 27 after the first participant to 95 after the fourteenth participant. No new final categories emerged after Participant 14; all remaining participants supported only categories that had already been generated. This situation is illustrated in Figure 15.

It is interesting that the general shape of the curve in Figure 15 closely resembles that of a charging capacitor in electronics with general function

$$f(t) = k(1 - e^{-\alpha t}).$$

When $t = 0$ in the function, $e^{-\alpha t} = e^0 = 1$, so $f(0) = 0$. As t approaches infinity, $e^{-\alpha t}$ approaches 0, so $f(t)$ monotonically approaches the maximum value k, slowing down at an exponential rate as it rises. This behavior occurs in capacitors because the initial charge entering a capacitor meets no resistance as there are no previously collected charges, but as charges accumulate in the capacitor, they increasingly repel new charges of the same polarity.

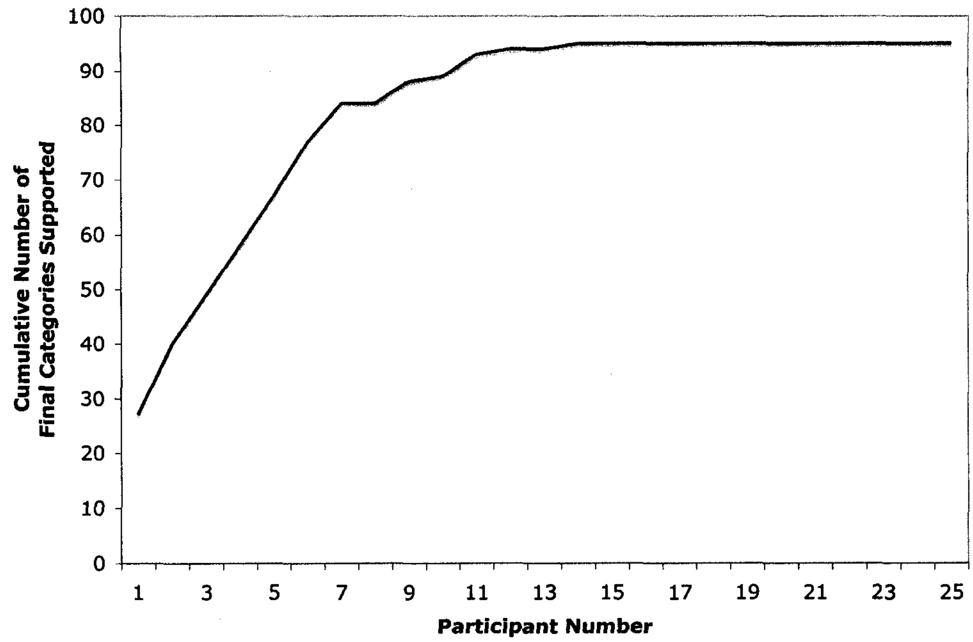

Figure 15. The cumulative number of final categories supported after each participant's data was analyzed.

In an analogous way, early participants were able to generate final categories at a high rate, but as final categories accumulated, it became more difficult for later participants to generate new final categories.

Axial Coding

Figure 16 illustrates the process of axial coding carried out for this study. As in Figure 10, the tiny text to the right of the flow diagram is there for illustrative purposes only; it is not meant to be read.

As categories emerged and evolved from the data analysis, I organized them into groups called supercategories by similarity of focus or theme. This extra level of organization is not common among grounded theory studies; however, most grounded theory studies have a much smaller number of categories due to a more narrow research focus than this explorational study. Organizing categories by supercategory allowed me to perform data analysis more effectively and to see the "big picture" in the data. Similar to categories, I examined the set of supercategories after the analysis of each participant's data to determine whether any could be combined or eliminated. The miniature ovals and boxes on the right side of Figure 16 give an example of how categories and supercategories could look.

Figure 17 shows that the number of supercategories decreased monotonically from 12 to 6, stabilizing after Participant 17. I did not identify supercategories by name until after Participant 9, revealing the increasing level of confidence that I had about the way data

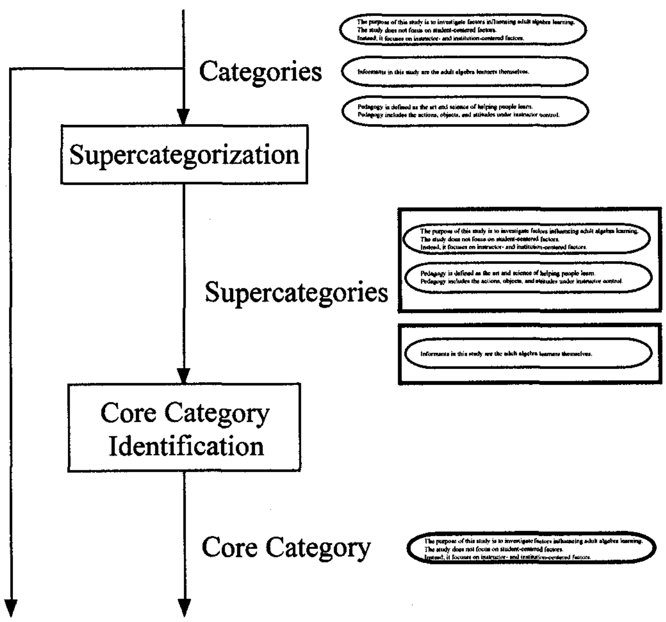

Figure 16. Axial coding identifies the supercategories and the core category of the study. (The tiny text is not meant to be read.)

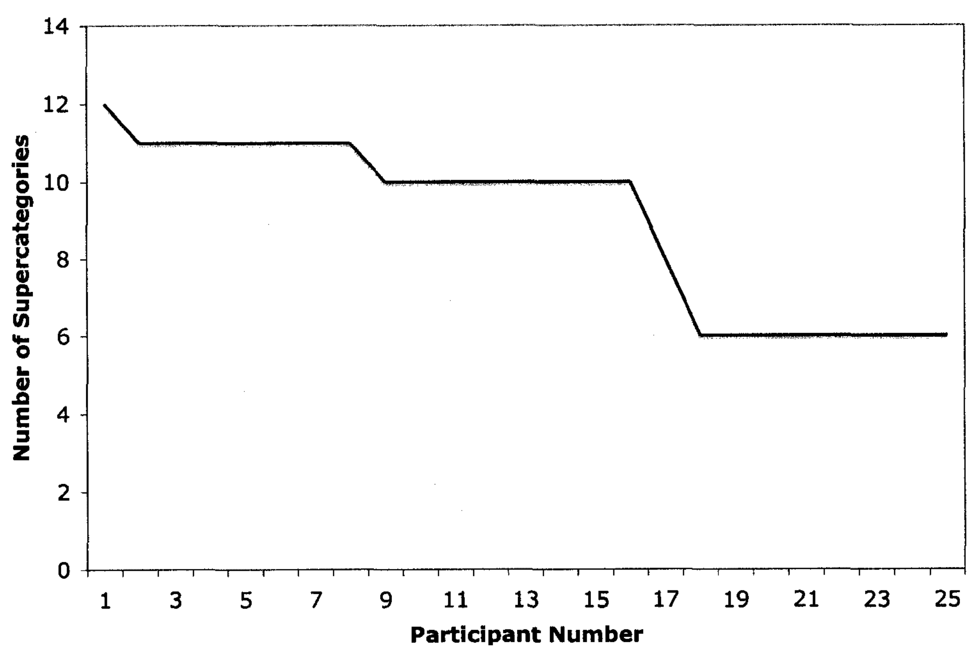

Figure 17. The number of supercategories after each participant's data was analyzed.

categorization was proceeding. The final supercategories and their descriptions are the following.

> **About Me:** 16 categories containing 259 tokens related to personal characteristics, feelings, and beliefs of students.
>
> **How I Learn:** 21 categories containing 330 tokens related to preferences and beliefs students have about learning.
>
> **About the Instructor:** 12 categories containing 152 tokens related to preferences and beliefs students have about instructors.
>
> **About the Course:** 19 categories containing 99 tokens related to preferences and beliefs students have about their algebra course, e.g., the way it is conducted, course requirements, exams, the textbook, etc.
>
> **About Algebra:** 17 categories containing 214 tokens related to beliefs and feelings students have about mathematics in general or specific areas of algebra or geometry.
>
> **Problems:** 10 categories containing 114 tokens related to student beliefs about or behavior exhibited during the written problems presented in the interview.

Saturation, the point at which additional participant data has little effect on category formation, is typically achieved after 20 to 30 participants in grounded theory studies (Creswell, 1998). Upon reaching saturation, axial coding is terminated and selective coding begins. The present study follows this pattern. The supercategories remained constant after Participant 17, and the categories remained constant after Participant 20. I terminated axial coding after Participant 25.

Figure 18 illustrates the number of tokens forming each final category. Categories are grouped by supercategory, as indicated by the labels just below the horizontal axis. Within each supercategory, categories are sorted from left to right in descending order by the number of participants who contributed tokens to the categories. Note that within each supercategory, column heights do not decrease monotonically. This is due to the fact that a column's height depends on the total number of tokens received by the category from all participants, but a column's left-right position within its supercategory depends on the number of participants who supported the category, i.e., provided at least one token for it. Thus, unusually tall columns for their positions may not have been popular categories among participants, but the participants who supported them did so adamantly. For example, the fifth category from the left in the supercatory **About the Instructor** is unusually tall for its position. Only 11 of 25 participants supported this category (provided at least one token for it); nevertheless, the category received a total of 34 tokens, well above the mean of 12.3 tokens per category. This category is "The way professors teach and their attitudes have a lot to do with student success; patience."

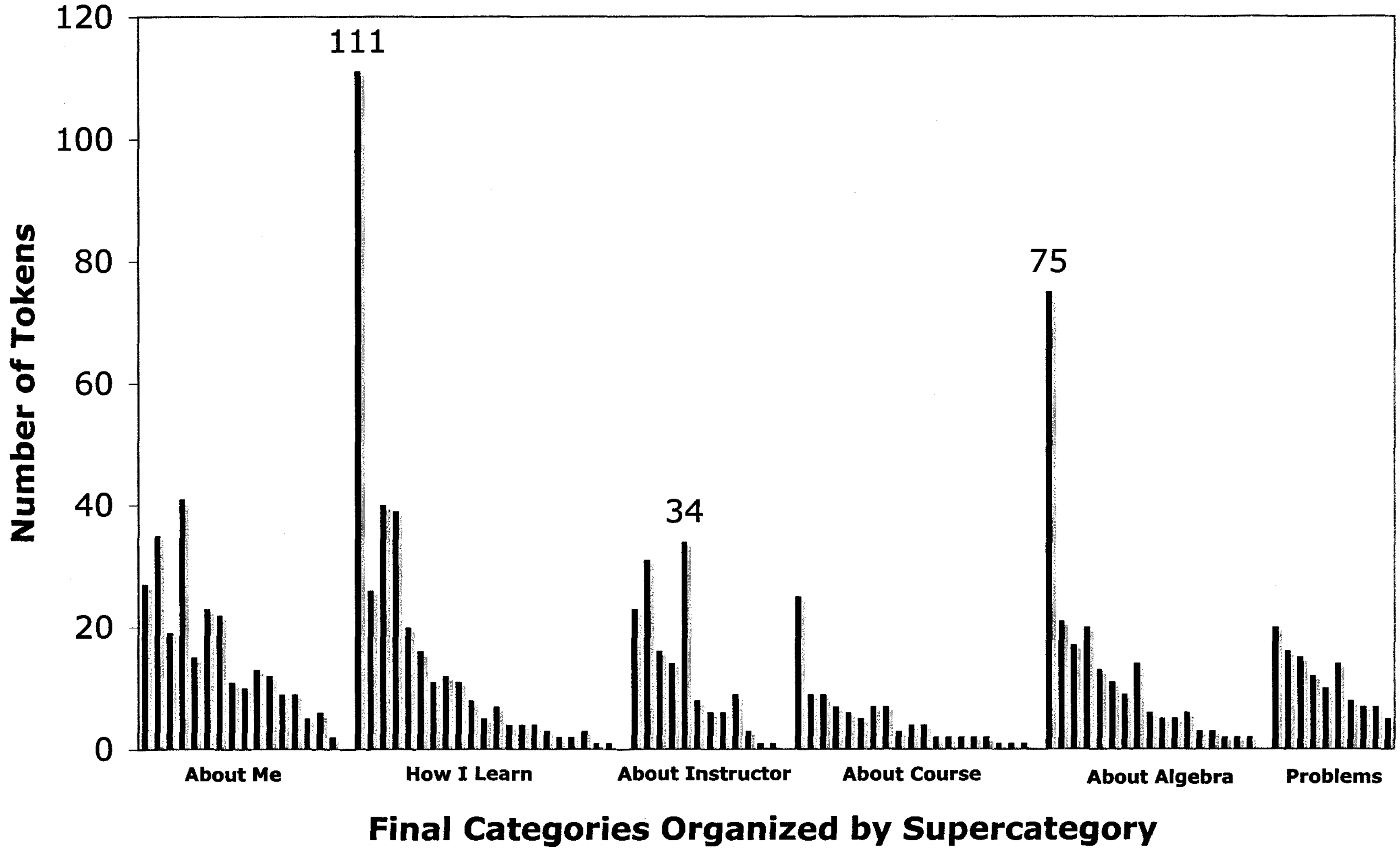

Figure 18. Number of tokens per category, grouped by supercategory and ordered within supercategories by number of supporting participants.

As seen in Figure 18, one category in the supercategory **How I Learn** garnered an exceptional number of tokens, 111, which was nearly 10% of all tokens generated by participants. This category was "If I can apply it to real life, it is easier to learn and remember." Of the 95 final categories, this was the only one to receive tokens from all 25 participants. In comparison, the next most popular categories received tokens from only 21 participants each. The category to receive the second highest number of tokens, or 75, was "Math takes a lot of time; there's too much information" in the supercategory **About Algebra**. This category received tokens from 20 participants. All other categories received fewer than 42 tokens each. Examples illustrating the spectrum of tokens contributing to the "If I can apply it to real life, it is easier to learn and remember" category follow.

- I would be more interested in algebra if it were more like what I do at work.
- I have to have a purpose. I have to see what I am going to use this for.
- What I like about algebra is that it helps you do things in real life.
- If I don't see the point of it, I won't remember it. I have to see why.
- I don't see the purpose of algebra.
- Why teach us things we'll never use?
- I like to first be exposed to information, then learn how it applies in real life, then practice applying it; that's how teaching should be.
- I have difficulty seeing how algebra will help me and what applications it has in the real world.
- I don't like accepting things on faith; I want to know now why they'll be useful to me.
- If I can apply it to real life, it is easier to learn and remember.
- If a topic has meaning to me, then I can learn it.
- I need to know what math will mean to me in my future; teachers should explain this more.
- I personally don't feel that algebra will be very useful to me.
- I like that algebra helps you do things in real life.

While some of the above tokens may not appear to fit precisely in the category "If I can apply it to real life, it is easier to learn and remember," the context in which each token was originally situated is not evident here. That context and the semantic content of each token were used together to determine the category for the token.

Because of its popularity among participants and its high level of token support, the category "If I can apply it to real life, it is easier to learn and remember" appears to be the core category of this study. The only other category to receive an exceptional number of tokens (see Figure 18) was "Math takes a lot of time; there's too much information." This second category counterpoints the core category nicely by offering a plausible explanation for the popularity of the latter, i.e., since math takes a lot of time and has so much information, it is easier to learn and remember if it can be applied to real life. The 85-item survey administered to triangulate data analysis results also supports “If I can apply it to real life, it is easier to learn and remember” as the core category, as this category was one of only eight items for which all 24 triangulation survey respondents (triangulators) were in agreement.

Selective Coding

Once the core category was determined, I began two analysis processes in parallel, selective coding and triangulation. Figure 19 illustrates the process of selective coding carried

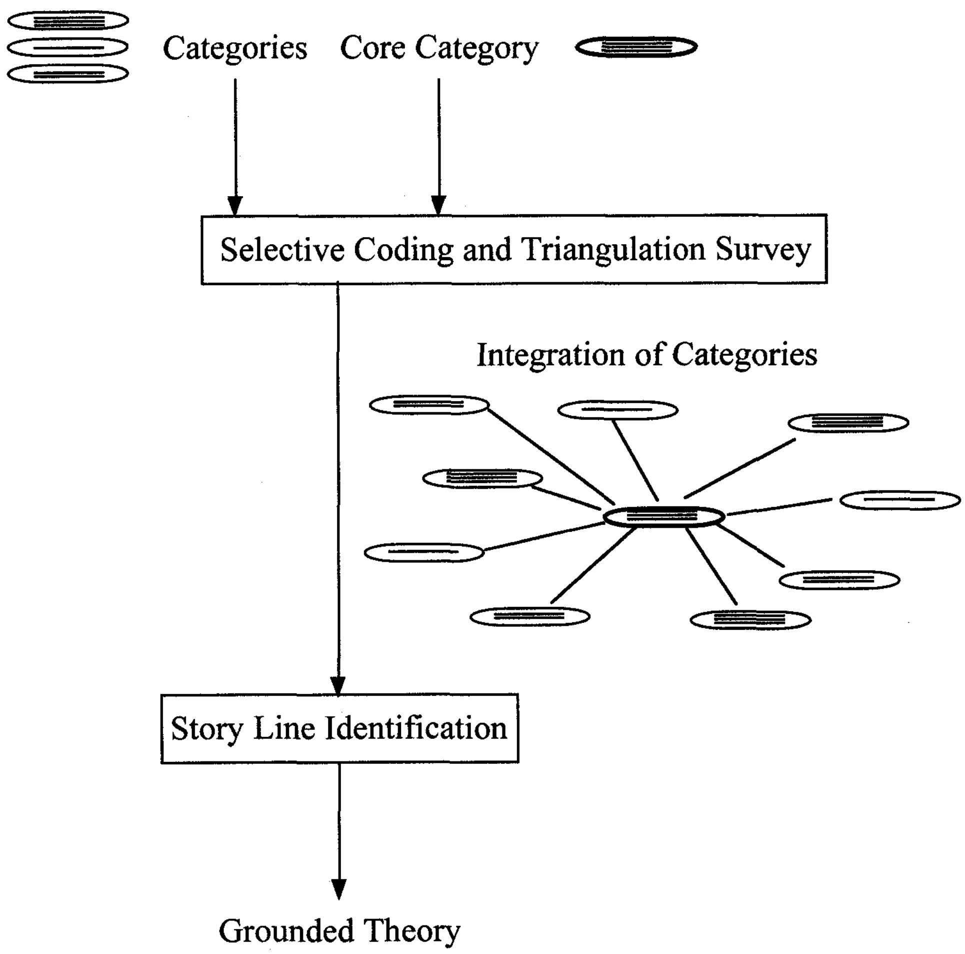

Figure 19. Selective coding relates the core category to other categories and identifies the story line.

out for this study. The next section discusses triangulation. As in previous coding flowchart figures, the miniature ovals near the flow diagram in Figure 19 give an example of how categories could appear.

The core category, "If I can apply it to real life, it is easier to learn and remember," is related to the supercategories, hence the categories themselves, as shown in Figure 20. The

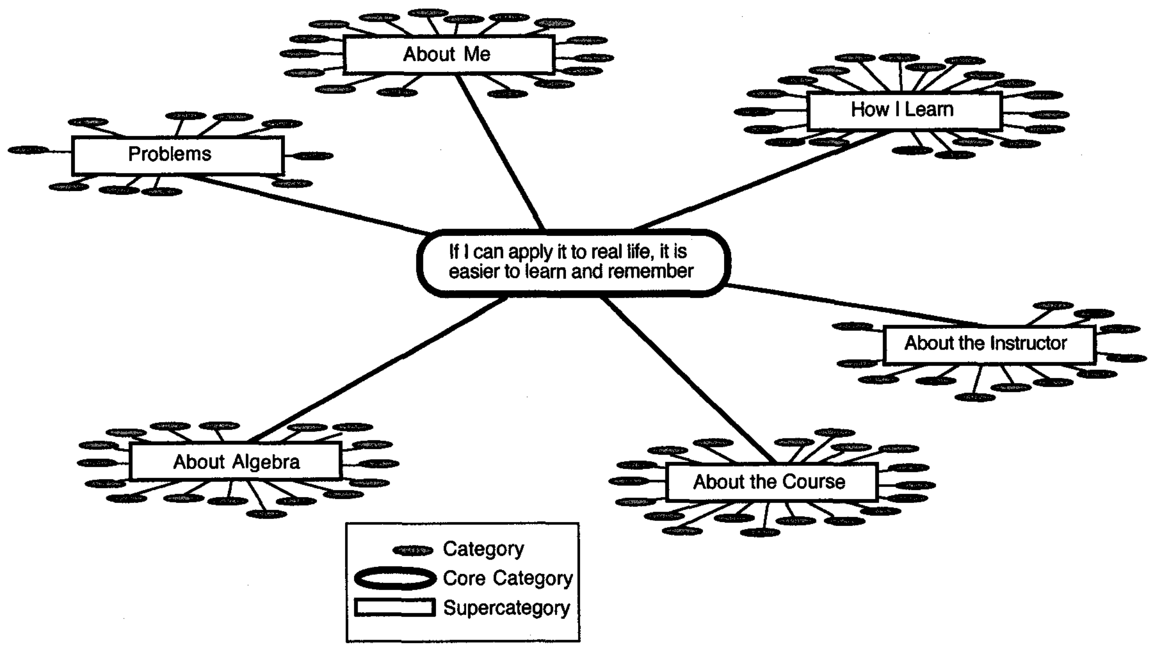

Figure 20. Integration of categories: The core category is related to each of the supercategories, hence to every category.

figure illustrates the particular integration of categories for this study. In the figure, the central bold oval contains the core category, and the outer rectangles contain the supercategories. Between 10 and 20 short line segments radiate out to tiny ovals from each supercategory rectangle. The tiny ovals represent the final categories of the study, each of which is connected to its corresponding supercategory. Note that the supercategory **How I Learn** contains 21 categories, but only 20 tiny ovals connect to it. This is because the core category in the center of the diagram is itself one of the categories belonging to the supercategory **How I Learn.**

The following list elucidates the relationships between the core category and the supercategories.

- The core category describes a belief held by algebra learners about themselves and is thus closely associated with the supercategory **About Me.**
- The core category describes how algebra learners believe they learn. It is closely associated with, and is contained in, the supercategory **How I Learn.**
- The core category does not refer explicitly to the instructor. Nevertheless, one of the major tenets of this study is that algebra instructors have the ability to positively influence their students' learning by respecting certain pedagogical factors in instructional design. This appears to be one such factor, i.e., instruction designed with

an eye toward real-world applications may facilitate algebra learning and remembering. Thus, the core category is related to the supercategory **About the Instructor**.

- As in the previous item, the core category does not refer explicitly to the course, but courses that emphasize real-world applications may facilitate algebra learning and remembering. Thus, the core category is related to the supercategory **About the Course**.
- The "it" of the core category refers to algebra, hence the core category is closely associated with the supercategory **About Algebra**.
- During the problem-solving portion of interviews, several participants vocally expressed cogent reasoning about the problems, even when they had trouble solving them. Such reasoning often took place as participants imagined themselves in an actual physical situation described by the problem. In the second problem, for example, various participants reckoned that there must be more students than professors at a college, hence the equation $6S = P$ could not be correct. Similarly, in the third problem, a few participants realized that a bicycle traveling downhill would move faster than one traveling uphill, hence the same distance would be covered in less time. This real-world-based metacognitive reasoning is evidence of a connection between the core category and the supercategory **Problems**.

The overall coding process for this study, consisting of open, axial, and selective coding, can be seen in the flow diagram of Figure 21. This figure, illustrating the constant comparison process particular to this specific study, is more detailed than the generic constant comparison process illustrated as part of Figure 9 on page 30.

Triangulation

Once the set of supercategories stabilized, I developed an anonymous survey for adult algebra students (see Appendix G) consisting of 85 Likert items derived from the categories of the first five supercategories. The 10 categories from the **Problems** supercategory were not included in the survey since survey respondents (triangulators) would not solve the written problems that participants solved during interviews. The purpose of this triangulation survey was to ascertain whether other algebra students agreed with categories generated by the participants, thereby affording a measure of confidence in the set of generated categories.

A total of 24 Mesa College students completed the triangulation survey during the spring 2006 semester. These triangulators were adults enrolled in Beginning Algebra (Math 95) or Intermediate Algebra (Math 96). In brief, the frequency with which triangulators agreed or strongly agreed with each category was at least as great as the frequency with which participants provided tokens for the category during interviews. In fact, at least one-sixth of triangulators agreed with each category. This supports the notion that the categories derived

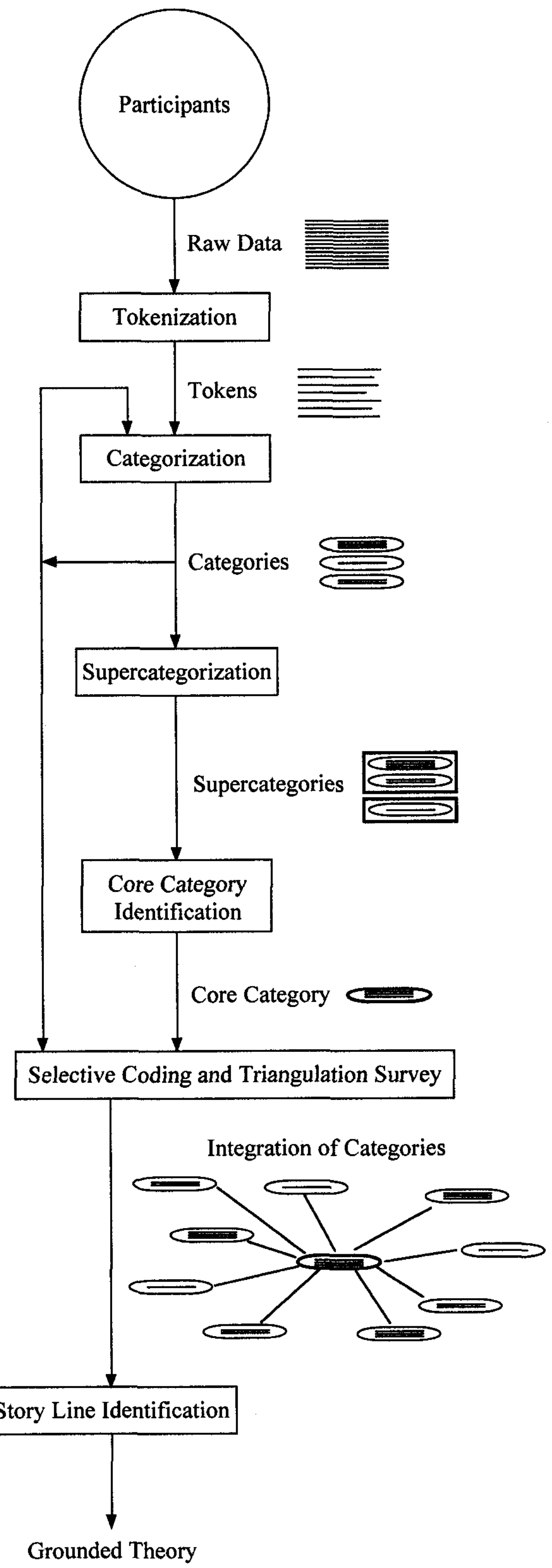

Figure 21. Detailed overall flow diagram of the constant comparison process for this study.

from participant interview data can yield valid inferences. The remainder of this section gives a more detailed description of triangulator and participant results for each supercategory.

Figures 22 through 26 show the percentage of participants who contributed tokens to categories in each supercategory and the percentage of triangulators who agreed or strongly agreed with those categories. For each category, the percentage of participants contributing at least one token to the category (participant support) is shown by a black bar and the percentage of triangulators agreeing or strongly agreeing with the category (triangulator support) is shown by a gray bar. As can be clearly seen in the five figures, categories within each supercategory have been sorted in descending order of participant support, i.e., by the lengths of the black bars. We also observe that triangulator support is never less than participant support for any category, i.e., no gray bar is shorter than the corresponding black bar. We must keep in mind, however, that triangulators merely identified and made decisions about categories, while participants generated the categories themselves. Participants would likely agree with many other categories besides those that occurred to them during interviews.

We can make the following general observations about the lengths of the black bars (participants) and gray bars (triangulators) in Figures 22 through 26.

- Categories for which both bars are fairly long are likely conspicuous topics of concern for many adult algebra learners.
- Categories for which both the black and gray bars are short may represent beliefs held by only a small number of adult algebra learners.
- Categories with short black bars (participants) and long gray bars (triangulators) may have more than one plausible explanation, including the following.
  - These categories may represent somewhat obscure ideas, *viz*, not many participants generated them; however, upon consideration, most adult algebra learners tend to agree with them.
  - These categories may represent topics that most adult algebra learners find important for learning, but they feel these topics are satisfied in their current learning situations.

Figure 22 shows the percentage of participants who contributed tokens to **About Me** categories and the percentage of triangulators in agreement with those categories. Participants and triangulators were both highly supportive of the first category, "Past experience aids my current learning; it makes things familiar; you get a practical understanding." This is consistent with the study's core category, "If I can apply it to real life, it is easier to learn and remember."

The levels of support for participants and triangulators differed significantly for several categories in this supercategory. For example, participants generated several categories that are generally considered to represent positive learning characteristics;

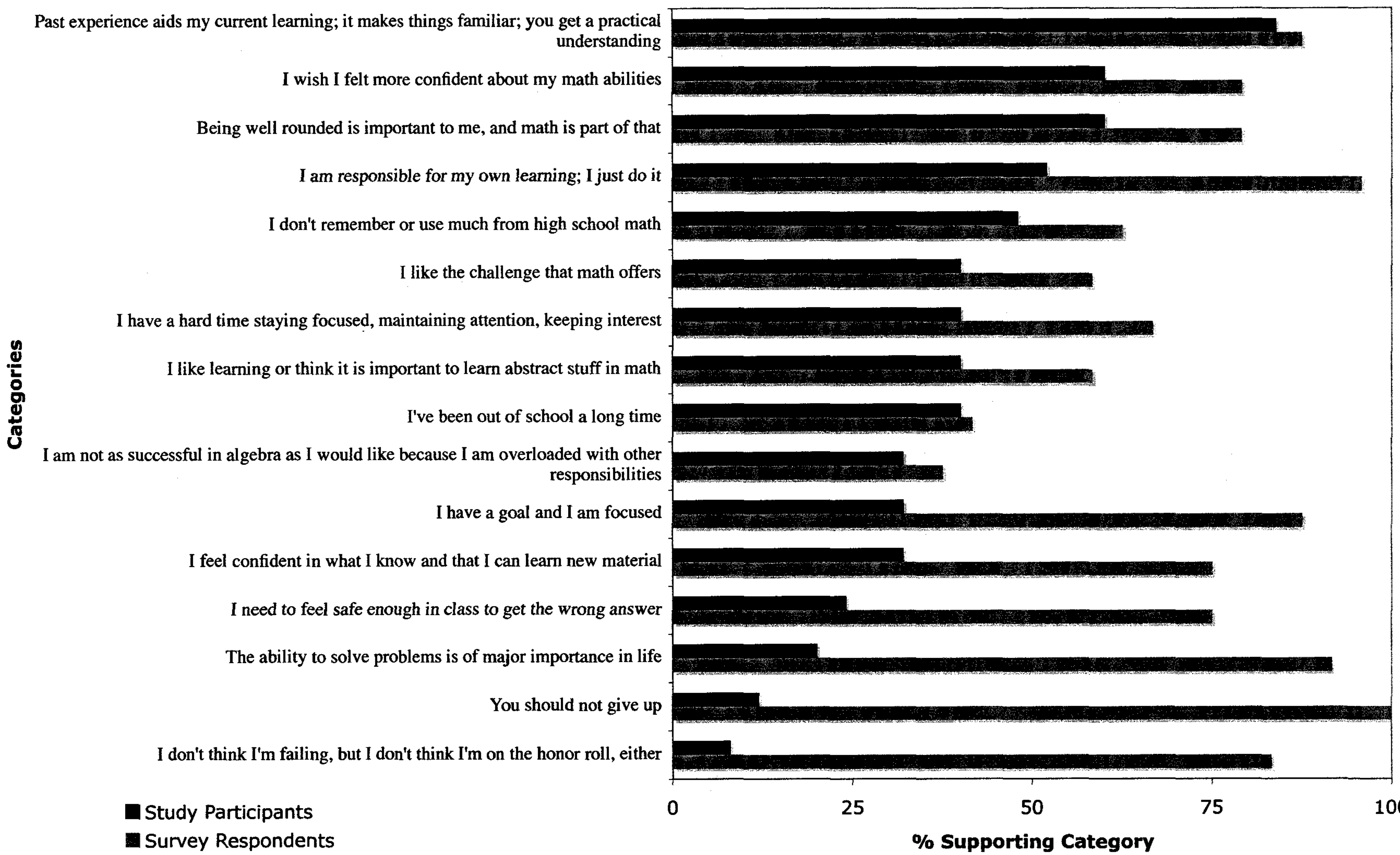

Figure 22. Participant and triangulator support for categories in the About Me supercategory.

however, these categories were supported by participants at much lower levels (10–50%) than by triangulators (75–100%). These categories include the following.

- I am responsible for my own learning; I just do it
- I have a goal and I am focused
- I feel confident in what I know and that I can learn new material
- The ability to solve problems is of major importance in life
- You should not give up

Perhaps many adult algebra learners have not personally adopted the above positive characteristics, but when queried about the characteristics, they find them reasonable. It is more likely that most adult algebra learners have these positive characteristics and simply take for granted that all reasonable adults have them, hence they may have failed to mention them during interviews.

Participants also supported two other categories in this supercategory at much lower levels (5–25%) than triangulators (75–80%), " I need to feel safe enough in class to get the wrong answer," and "I don't think I'm failing, but I don't think I'm on the honor roll, either." It may be that most adult algebra learners already find their classes emotionally safe, but at the same time they realize such safety is an important aspect of the learning environment. Also, it appears that most adult algebra learners may not know how well they are doing in their classes, although some may not be concerned by this lack of knowledge.

Figure 23 shows the percentage of participants who contributed tokens to **How I Learn** categories and the percentage of triangulators who agreed with those categories. The first category in this supercategory, "If I can apply it to real life, it is easier to learn and remember," is the core category, having received 100% support from both participants and triangulators. The second category in this supercategory, "Making mistakes is fundamental to learning; they make me aware of my own shortcomings" was also highly supported by both participants and triangulators. This category acts rather like a semantic bridge between the core category and the highly supported first category of the supercategory **About Me**, "Past experience aids my current learning; it makes things familiar; you get a practical understanding." In other words, participants and triangulators seem to be saying that making mistakes (and presumably correcting or at least analyzing them) provides a form of practical, real-life experience that can facilitate learning and remembering.

Another category in the supercategory **How I Learn** received only 50% support from participants but 100% support from triangulators. This category, "I need to see and try things in order to understand them," is semantically related to the other highly supported categories of the first two supercategories.

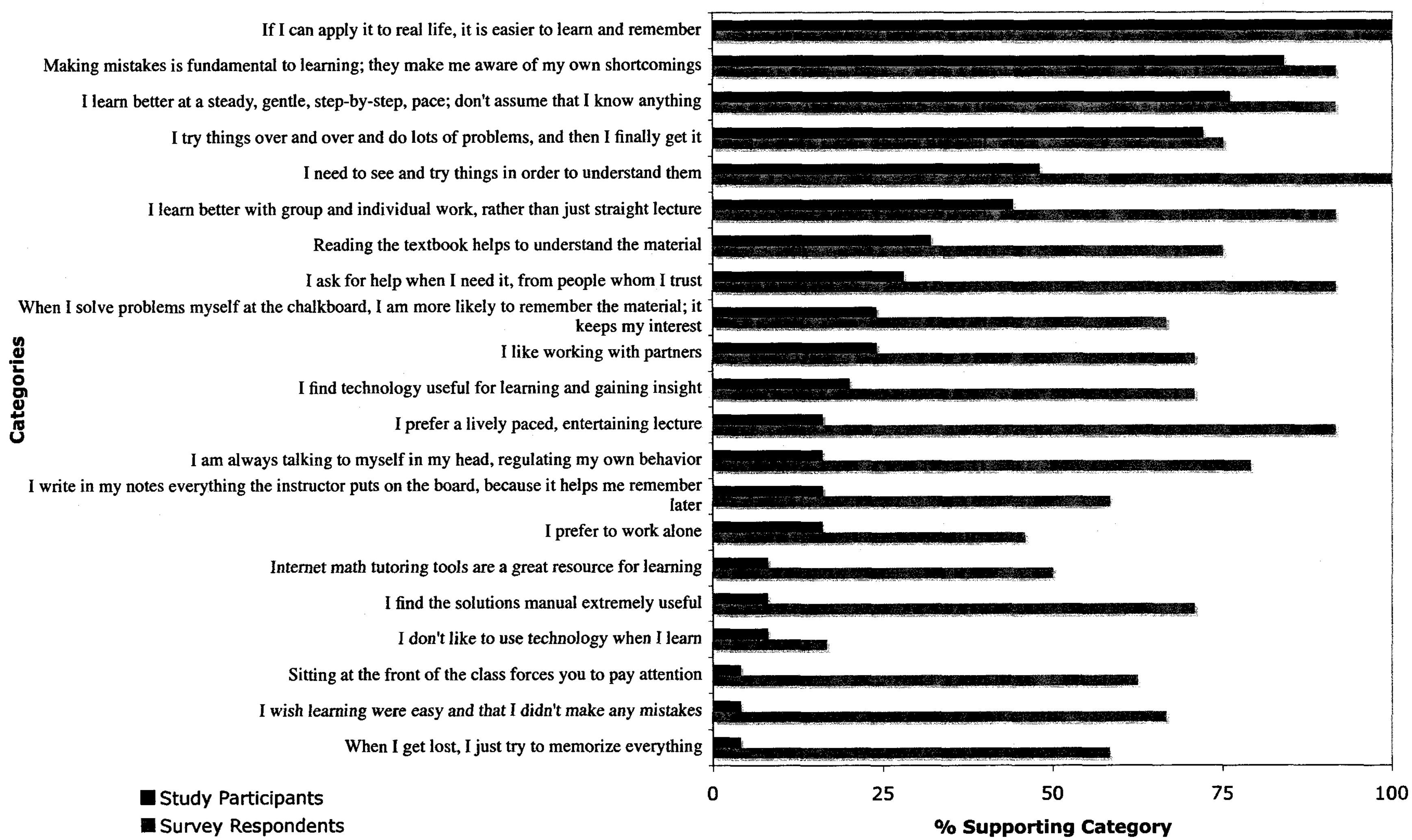

Figure 23. Participant and triangulator support for categories in the How I Learn supercategory.

Two categories in this supercategory deal with the use of technology in learning. It is reassuring to see the consistency revealed by the fact that about 71% of triangulators agreed with the pro-technology category "I find technology useful for learning and gaining insight," while only about 17% of them agreed with the anti-technology category "I don't like to use technology when I learn." On the other hand, it is curious that 71% of triangulators stated they like working with partners, while 46% stated they prefer to work alone. Perhaps some adult algebra learners prefer to work alone, but they still enjoy working with partners some of the time.

Notably, the last category in this supercategory, "When I get lost, I just try to memorize everything," was generated by only 5% of participants but supported by 60% of triangulators. Perhaps most adult algebra learners tend not to get lost in class, but when they do, rote memorization is their fail-safe.

Figure 24 shows the percentage of participants who contributed tokens to **About the Instructor** categories and the percentage of triangulators who agreed with those categories. The following categories were among those of this supercategory that received medium participant support (50–80%) but high triangulator support (85–100%).

- I prefer that the instructor takes care of all aspects of planning the course; they're the experts
- I would like to know as much about instructors' teaching styles and policies as possible before I sign up for their classes
- The way professors teach and their attitudes have a lot to do with student success; patience

Students seem to be saying that instructors should be allowed to design and conduct courses as they see fit, without external influence from administrators or the students themselves; however, students would like to know as much as possible about those courses before they register for them. In this way, they can find a good fit for their learning styles and personalities, thereby improving their chances of success.

Other categories in this supercategory received mediocre participant support (15–65%) but high triangulator support (75–100%). These categories seemingly reveal certain characteristics that most adult algebra learners want to see in their instructors, including patience, well-organized and clear writing and speech, and flexibility in policies and teaching methods. Students also appear to desire personal attention from their instructors and active participation from everyone in the class.

The category "I want my instructor to show me how to do things in algebra and explain to me why they work" received only 30% participant support but 100% triangulator support. This is consistent with two other highly supported categories (50–75% by

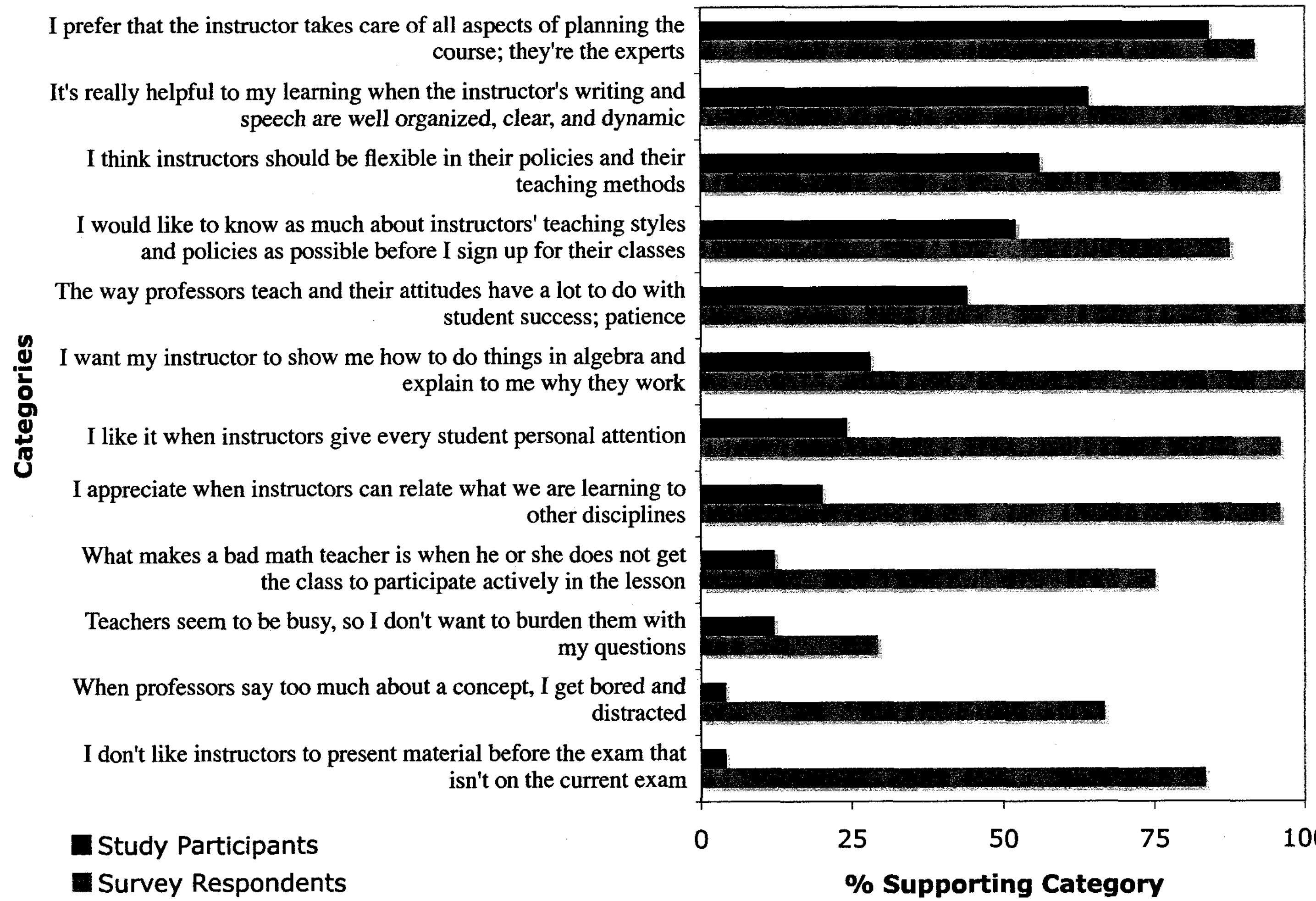

Figure 24. Participant and triangulator support for categories in the About the Instructor supercategory.

participants, 75–100% by triangulators) in the supercategory **How I Learn**, "I try things over and over and do lots of problems, and then I finally get it," and "I need to see and try things in order to understand them." That is, adult algebra learners seem to believe that they will learn algebra by doing it. This practical application approach to learning is supported by the core category and by the highly supported category (20% by participants, 95% by triangulators) "I appreciate when instructors can relate what we are learning to other disciplines" in this supercategory.

Figure 25 shows the percentage of participants who contributed tokens to **About the Course** categories and the percentage of triangulators who agreed with those categories. Adult algebra students apparently believe (35–60% of participants, 95–100% of triangulators) that doing homework and receiving regular and timely feedback are essential for their success in algebra. This is consistent with the practical application approach to learning of the previous paragraph. However, it appears somewhat inconsistent with one fairly highly supported category (10% of participants, 50% of triangulators) from this supercategory, "Exam problems should test technical skills; they shouldn't be applied or word problems." In fact, a small proportion of adult algebra learners (20% of participants, 35% of triangulators) prefer not to have exams at all, but the majority (5–10% of participants, 70–85% of triangulators) seem to believe that tests and quizzes motivate them and keep them on track.

Two highly supported categories (20–30% of participants, 80–90% of triangulators) in this supercategory support fundamental assumptions of our current system of higher education, "Lecture is an essential part of the learning process," and "Missing class is devastating to learning." At the same time, the highly supported category (10% of participants, 75% of triangulators) "Sometimes I am so confused that I don't even know what to ask" may be an indication that our current system has room for improvement.

This study provided an anonymous forum for adult algebra learners to vocalize their feelings and thoughts about their courses and instructors. It is interesting that a few participants (10–20%) and several triangulators (60–90%) commented on their fellow students, agreeing that "Some students are not prepared or serious and should not be in our class," and "Some people need to work harder to learn instead of complaining about the class."

Figure 26 shows the percentage of participants who contributed tokens to **About Algebra** categories and the percentage of triangulators who agreed with those categories. The first two categories in this supercategory, both highly supported by participants (60–80%) and triangulators (85–100%), bring to light what is perhaps the major concern that adult algebra learners have with the subject: there is too much to remember. This is also hinted at by the core category, "If I can apply it to real life, it is easier to learn and remember." Mathematics instructors might respond to this concern by stating that math should be understood, not

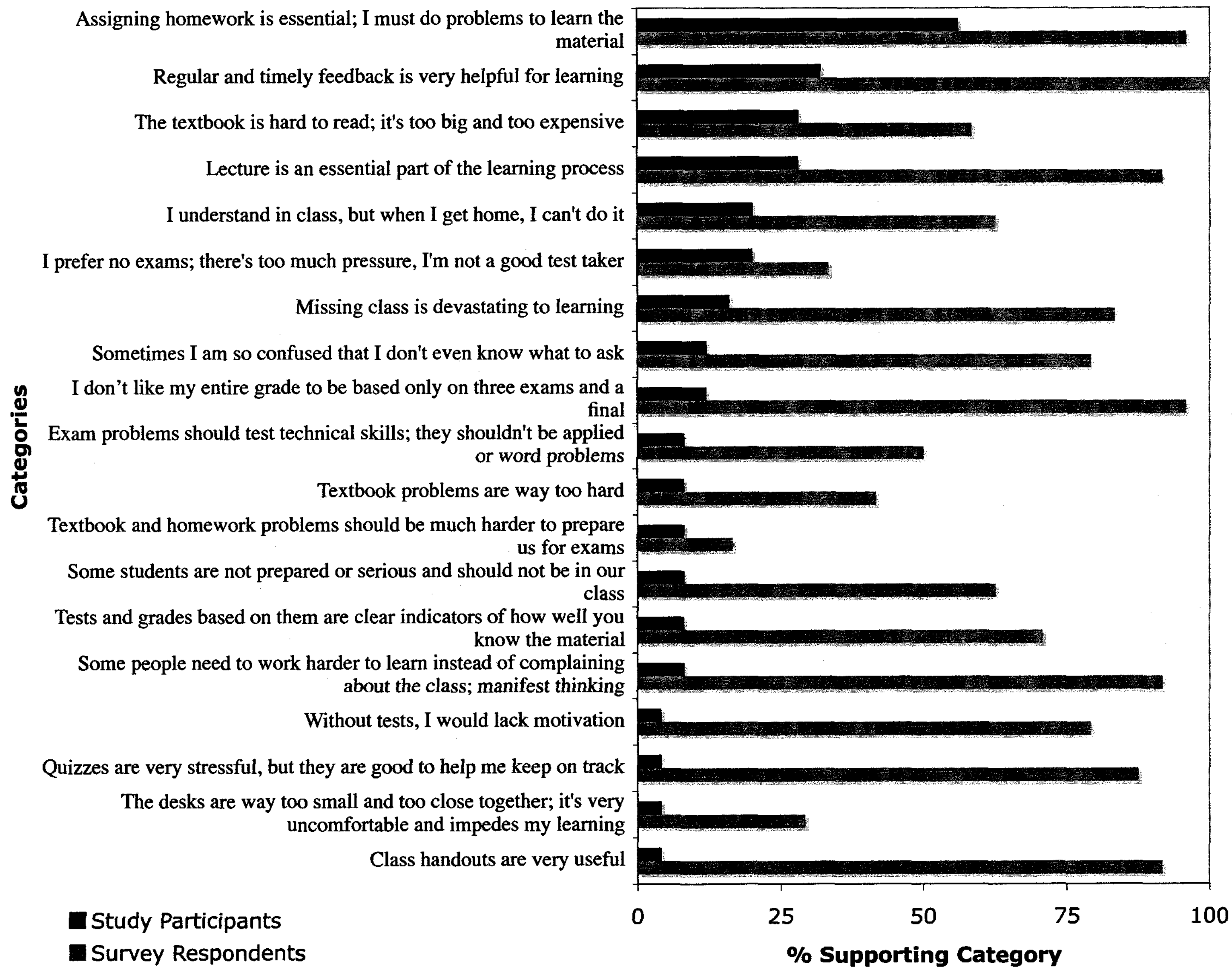

Figure 25. Participant and triangulator support for categories in the About the Course supercategory.

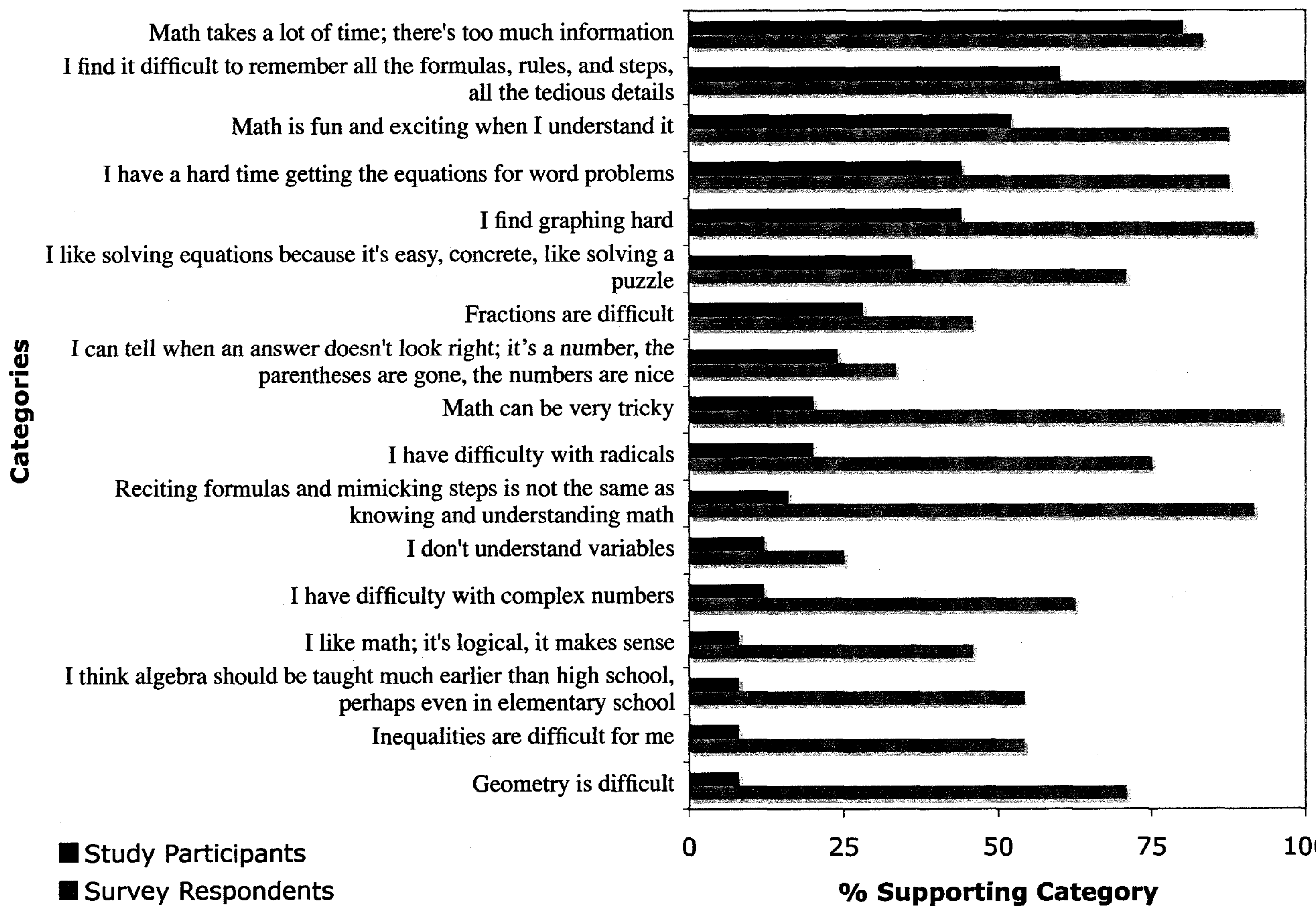

Figure 26. Participant and triangulator support for categories in the About Algebra supercategory.

memorized. Interestingly, adult algebra learners seem to agree, as evidenced by their support (10–15% participants, 45–90% triangulators) of the two categories "Reciting formulas and mimicking steps is not the same as knowing and understanding math" and "I like math; it's logical, it makes sense." On the other hand, 20% of participants and 95% of triangulators agreed that "Math can be very tricky." One participant even contributed the following token to this category: "Tricking us is all part of math, and they get me every time."

Categories in this supercategory uncover several areas of mathematics with which adult algebra learners tend to struggle, including getting equations for word problems (45% of participants, 85% of triangulators), graphing (45% of participants, 90% of triangulators), radicals (20% of participants, 75% of triangulators), complex numbers (15% of participants, 60% of triangulators), inequalities (10% of participants, 55% of triangulators), and geometry (10% of participants, 70% of triangulators). A small proportion (10% of participants, 25% of triangulators) even stated that they don't understand variables, which ironically are often considered the heart of algebra. Nevertheless, many adult algebra learners (60% of participants, 85% of triangulators) have apparently experienced joy in learning algebra, agreeing that "Math is fun and exciting when I understand it."

FINDINGS AND RESEARCH HYPOTHESES

This section discusses the results of data analysis as they pertain to each research hypothesis.

Hypothesis 1: Certain Pedagogical Factors Significantly Influence Adult Algebra Learning

The first hypothesis of this study is that certain pedagogical factors at Mesa College have a significant influence on the learning of algebra by many adult students. Categories generated by participants confirm this hypothesis. In fact, about 80% of the final categories were related to pedagogical factors that may influence learning. The categories suggest the existence of four classes of pedagogical factors that influence adult algebra learning: (1) instructional style and policies, (2) course activities, (3) learning aids, and (4) course pacing. Furthermore, a few characteristics emerged that appear to be common among adult algebra learners and may have a significant influence on learning. Each of these is discussed below.

INSTRUCTIONAL STYLE AND POLICIES

Adult algebra learners appear to strongly believe that the way professors teach has a lot to do with student success. In particular, adult algebra learners believe that they are much more successful with patient instructors. Furthermore, they would like instructors to exhibit flexibility in their policies and teaching methods. Adults want to learn algebra in an emotionally safe environment where they are free to make mistakes without feeling humiliated. They want personal attention from the instructor; in fact, they believe that good instructors are those who engender active participation from everyone in the class. Adults recognize the instructor's right to design and implement a course as he or she sees fit; however, they would like to know as much as possible about the course's design and implementation before committing themselves to it.

Course Activities

Adult algebra learners agree that lecture is an essential part of learning; however, they prefer this not be the only mode of teaching. They feel that they learn better if they also have group and individual in-class work, including solving problems at the chalkboard themselves. Adults overwhelmingly agree that it is essential for instructors to assign and grade homework, for two major reasons. First, adults have a need to see and try things, sometimes over and over, in order to understand them. Second, adults want to know how they are doing in the class by receiving regular and timely feedback from their instructor. Beyond homework, adults believe that exams are an important part of learning, mainly as a tool to motivate learning, but also to receive feedback on how well they are learning the material. Adults prefer several smaller exams to a few large ones during the semester, as this gives them frequent feedback on their learning and insulates their grade in case of a single poor test performance.

Learning Aids

Some adult algebra learners believe that educational technologies such as graphing calculators, computer software, and internet math tutors are great tools for learning, while others prefer to avoid them and rely solely on class instruction, reading the textbook, and checking their problem solving work with the solutions manual. Most adults find handouts from their instructors useful. Perhaps the most important thing that adults believe instructors can do to help them learn is to relate what they are learning to other disciplines or to something they already know from past experience. Adult algebra learners vehemently believe that if they can apply something to real life, then it will be easier to learn and remember.

Course Pacing

Although it appears contradictory, adults seem to simultaneously prefer learning algebra at a lively pace and at a gentle, step-by-step pace. Many adult algebra learners tend to become bored or distracted when instructors say too much about a concept. They want lectures to move along at a steady clip without dwelling too much on any single idea. Most adult algebra learners find it easier to learn when their instructor's speech and writing are well organized, clear, and dynamic.

Virtually all adult algebra learners agree that math takes a lot of time due to the vast amount of information to be learned. In their own words, they find it difficult to remember all the formulas, rules, steps, and tedious details. They seem to want to learn only what is essential. Most adult algebra learners prefer that instructors not present material before an exam that won't appear on the exam. Certain topics in algebra courses are especially troublesome for adult learners and may merit extra instructional time. These include graphing, fractions, radicals, complex numbers, inequalities, and geometry. Some adults even report difficulty understanding variables, the most fundamental of algebraic concepts.

Characteristics Shared by Adult Algebra Learners

Certain non-pedagogical characteristics emerged during interviews that may have an effect on adult algebra learning. For example, most adult algebra learners appear to be quite emotionally mature. They believe that knowing mathematics will make them more well rounded and will help them become better problem solvers, both of which they consider important in life. Virtually all of them feel responsible for their own learning and believe that they must be persistent. Many feel confident in their ability to learn new material, and most have a habit of self reflection, allowing them to observe and regulate their own behavior. Many even observe their peers and comment on those who are not prepared or serious enough to be in their class, or those who need to work harder instead of complaining about the class. Even though not many adult algebra learners claim to like math, most find math fun and exciting when they understand it.

Hypothesis 2: Adults Have the Same Algebra Misconceptions as Children

The second hypothesis of this study is that many adult students have similar misconceptions when they learn algebra to those held by children who are learning algebra. Specifically, they exhibit the following misconceptions: *expression as procedure*, *letter as label*, and *graph as path*. The data appear to partially support this hypothesis. Figure 27 compares children and adult rates for the three misconceptions examined in this study. While adults seem to share the *letter as label* misconception (see Figure 28) and the *graph as path* misconception (see Figure 29) with children, unlike children they appear to be free of the *expression as procedure* misconception. The adult misconception rates (80% and 60%) are slightly higher than the child rates (67% and 50%) for the two misconceptions that they share

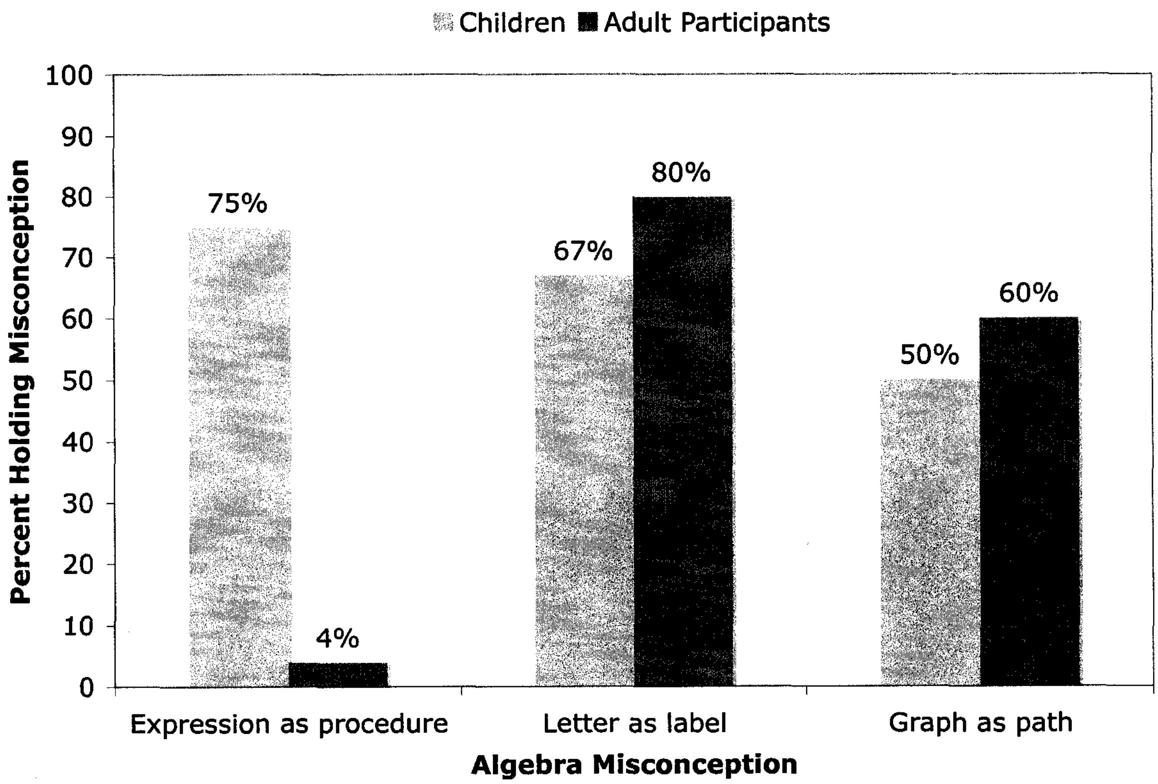

Figure 27. The percentage of children and of adult participants holding each algebra misconception.

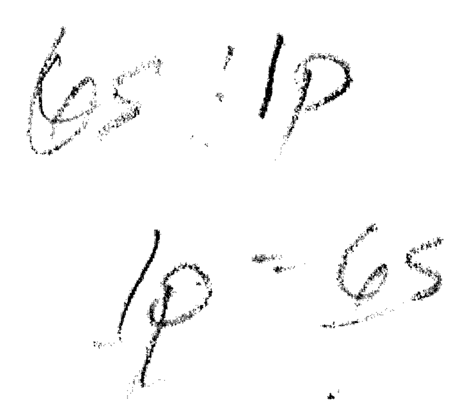

Figure 28. An anonymous participant's written response exhibits the *letter as label* misconception. Participants were asked to use variables S and P to represent the situation "Mesa College has six times as many students as professors."

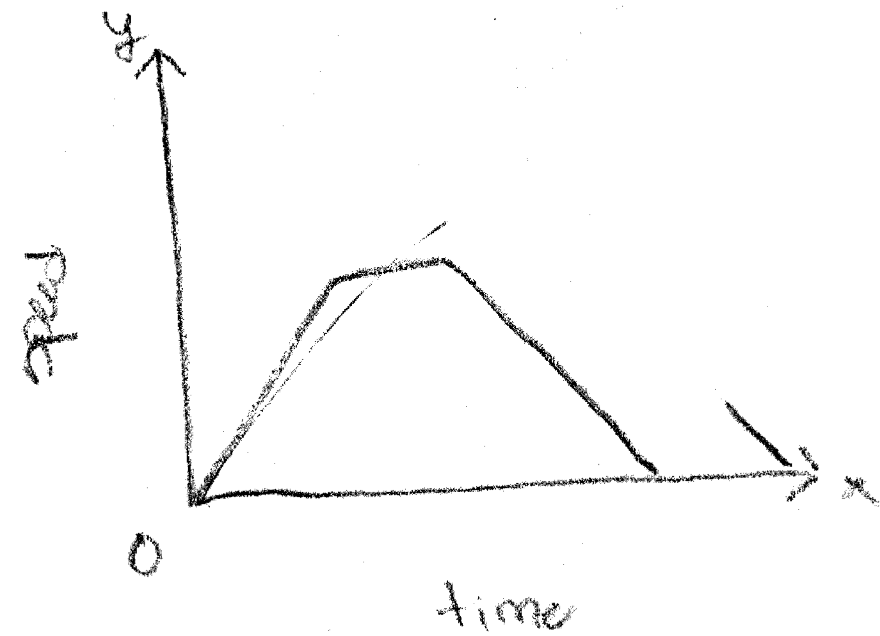

Figure 29. An anonymous participant's written response exhibits the *graph as path* misconception. Participants were asked to sketch speed vs time for a bicycle ascending then descending a steep hill.

with children. Perhaps this is due to bias in the sample, e.g., children who hold algebraic misconceptions and never resolve them may tend to become adults who need to learn algebra.

The adult misconception rate for the *expression as procedure* misconception was only 4%, compared to 75% for children. The reason for this extreme drop in rate may be that adults are able to think more abstractly about symbolic expressions than children. Unlike children, adults may not feel compelled to perform explicit operations appearing in expressions. On the other hand, it may be that adults have simply been conditioned over the years to accept answers that are not purely numerical.

Hypothesis 3: Adult Learning Characteristics Should Guide Curriculum Development

The third hypothesis of this study is that adults learn algebra more successfully when general adult learning characteristics guide curriculum development. These characteristics include the five specified by Knowles in his theory of andragogy—self-concept, experience, readiness to learn, time, and orientation to learning—as well as academic fossilization and metacognition. This hypothesis appears to be well supported by the data of this study.

SELF-CONCEPT

Self-concept describes a person's self-directedness, ability for self-instruction, and responsibility for personal decisions. Participants in this study exhibited personal responsibility to a large degree, as evidenced by the following final categories.

- I am responsible for my own learning; I just do it
- I feel confident in what I know and that I can learn new material
- You should not give up
- I am always talking to myself in my head, regulating my own behavior
- Some students are not prepared or serious and should not be in our class
- Some people need to work harder to learn instead of complaining about the class; manifest thinking
- Reciting formulas and mimicking steps is not the same as knowing and understanding math
- I ask for help when I need it, from people whom I trust

On the other hand, the data also indicate that self-concept may not be completely developed in all adults. Many adult algebra learners simply try to memorize everything when they feel lost. Virtually all believe that an instructor's teaching method and attitude have a lot to do with student success, indicating a potential external locus of control. A small proportion feel that they will be a burden to their instructor if they ask too many questions.

EXPERIENCE

According to Knowles, adults need to learn from experience. The data support this. The majority of participants agreed that past experience aids their current learning by making things familiar and giving them a practical understanding. They also concurred that making mistakes is fundamental to learning. Many adult algebra learners apparently need to see and try things over and over before they achieve understanding.

READINESS TO LEARN

Readiness to learn refers to an adult's ability to absorb new information at a specific time of life. Knowles claimed that for adults this is regulated by the demands imposed by social roles. That is, before adults are ready to learn something new, they need to believe that it will help them perform some task or fulfill some duty. The data of the study wholly support this idea. The study's core category, "If I can apply it to real life, it is easier to learn and remember," suggests that adult algebra learners want and are most able to learn what they need in order to satisfy their real-life responsibilities. They wonder why instructors teach them things they will never need; indeed, the majority claim that they do not remember or use much from high school math. Almost all adult algebra learners believe that the ability to solve problems is of major importance in life, and they feel that they have a goal and are focused in their studies. Even so, many assert that they have a hard time maintaining focus, attention, and interest, perhaps because they are overloaded with other responsibilities.

TIME

Knowles proposed that the time between what adults learn and its application should be short. In other words, adults prefer what they are learning to have immediate value and application in their lives. As for the previous characteristic, the core category of this study ("If I can apply it to real life, it is easier to learn and remember") strongly supports this. Virtually all adult algebra learners want their instructors to show them how to do things and explain to them why they work, and most appreciate instructors who can relate what they are learning to other disciplines.

ORIENTATION TO LEARNING

Knowles' concept of orientation to learning refers to a preference for the practical over the abstract, for problem solving over the passive acquisition of abstract concepts. The data indicate that adult algebra learners tend to prefer concrete, practical, problem solving approaches to learning. Adult algebra learners generally believe that past experience aids their current learning by making things familiar, giving them a practical understanding. The core category suggests that most adults learn practical information and skills easier than abstract concepts. It is interesting that most adult algebra learners believe that the ability to solve problems in life is extremely important, yet around half of them appear to prefer exams that test technical skills only, rather than the ability to solve applied problems. Not all adults learn algebra for purely practical reasons. At least half enjoy the challenge math offers and think it is important to learn abstract mathematical concepts. Perhaps they believe that this type of math will help them fulfill their goal of becoming more well rounded.

ACADEMIC FOSSILIZATION

Academic fossilization, or the misapplication of previously learned skills, was detected in about 28% of participants during the problem solving phase of interviews. This was most commonly seen when participants performed an incorrect distribution in the first problem, converting $1 + (x + yz)$ into $1x + 1yz$ (see Figure 30). It was also observed when participants incorrectly combined unlike terms, as in $x + yz = xyz$.

METACOGNITION

Metacognition is the ability of individuals to be aware of and regulate their own knowledge and learning. Over 50% of participants exhibited metacognition during interviews.

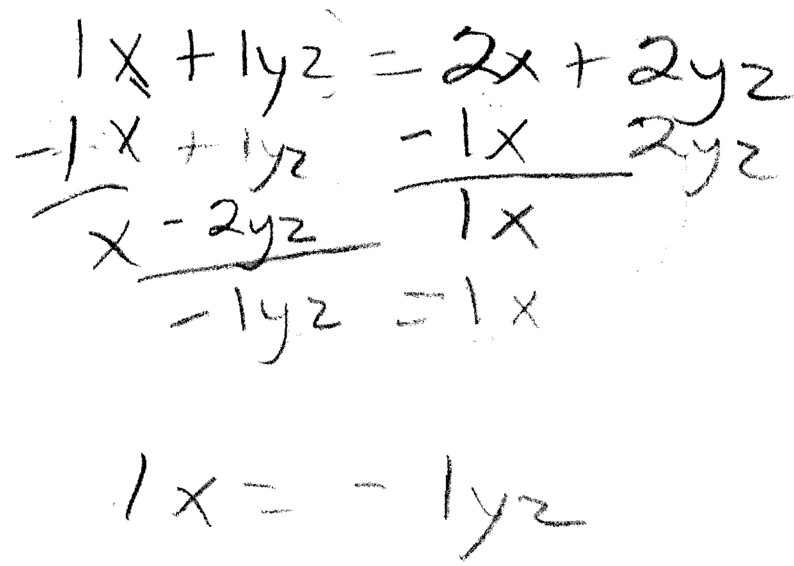

Figure 30. An anonymous participant's written response exhibits academic fossilization. Participants were asked to solve the equation $1 + (x + yz) = 2(x + yz)$ for x. This example may also illustrate the *expression as procedure* misconception.

Several participants reflected on their work as they solved written problems in interviews, for instance, by

- Checking their answers to the first problem, $1 + (x + yz) = 2(x + yz)$
- Realizing that $6(12) \neq 2$, so $6S = P$ is not right in the second problem
- Observing that there must be more students than professors in the second problem
- Labeling axes in the third problem with reasonable ranges of values
- Recognizing in the third problem that when a bicycle descends a hill, it does so quickly and therefore takes up less time on the horizontal axis than it does as it ascends the hill.

A large proportion of participants and triangulators also stated explicitly that they employ self-reflection to modify their own behavior.

Other Adult Learning Characteristics

Besides the anticipated adult learning characteristics of self-concept, experience, readiness to learn, time, orientation to learning, academic fossilization, and metacognition, two other characteristics emerged from the data that appear common to most adult algebra learners. These were (1) concerns over limited memory for learning, and (2) a need for feedback. It appears that most adults think algebra is difficult to learn because it requires them to remember too much information, i.e., too many formulas, rules, steps, and other tedious details. Adults often prefer that their algebra instructors make no assumptions about their prior algebraic knowledge; rather, they would like their instructors to conduct courses at a steady, gentle, step-by-step pace, starting at the very beginning of the subject. Most adult algebra learners wish they felt more confident in their mathematical abilities, although many do appear to feel confident in what they know and that they can learn new material. Most adult learners appear to not know where they stand in their algebra class; in point of fact, they believe that regular and timely feedback would be very helpful for their learning.

FINDINGS AND DEMOGRAPHICS

This section discusses the results of data analysis as they pertain to the demographics of the sample. Recall that a participant supported a category if he or she generated at least one token belonging to the category. The level of support of a group for a category is the proportion of group members who supported the category. Category support data from interviews were organized in three ways, (1) by gender (female or male), (2) by ethnic group (Asian, Black, Latino, or White), and (3) by age group (18–25, 26–35, or 36–48). Kruskal-Wallis nonparametric tests were employed to determine whether groups exhibited significant differences in the level of support they gave to each category. The results of the Kruskal-Wallis tests at the $\alpha = 0.05$ level are described in the following three sections.

Category Support and Gender

The sample was divided by gender as shown in Table 2. Figure 31 illustrates the

Table 2. Sample by Gender

Gender	Number of Participants
Female	12
Male	13

percentages of female and male students at Mesa College (outer ring) and in the sample (inner ring). Notice that there is relatively close agreement between the population and the sample percentages.

Figure 32 shows the percentage of participants of each gender who supported each category. Subcharts in the figure correspond to the supercategories named in the subcharts' titles. Kruskal-Wallis analysis of this data revealed only two categories with significant differences among support levels. The two categories, from the **How I Learn** and **About Algebra** supercategories, were the following.

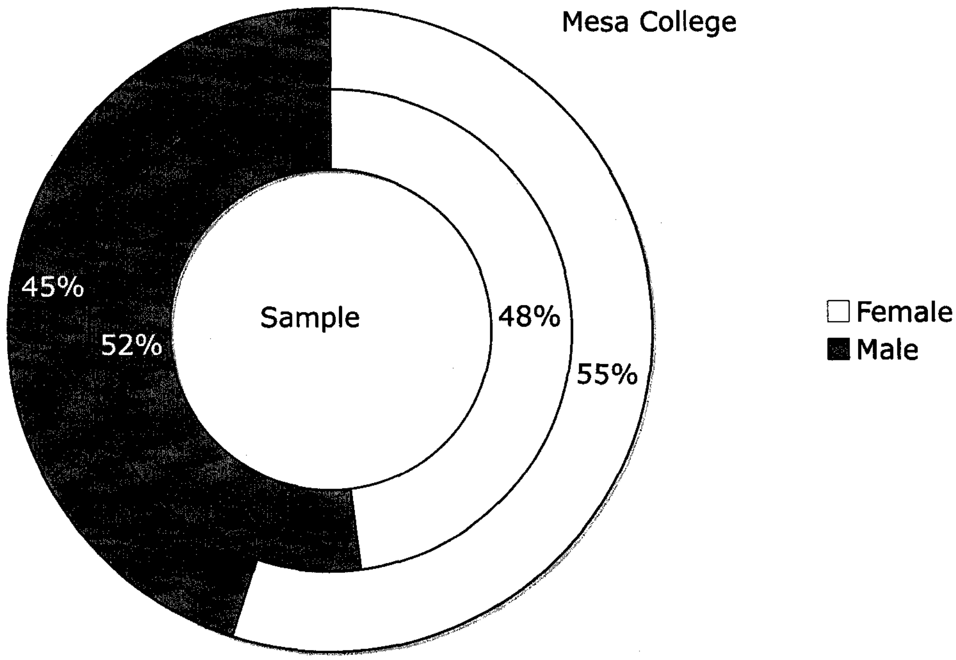

Figure 31. Breakdown by gender of the population of Mesa College students (outer ring) and of the sample (inner ring). The sample is fairly representative of the population for gender.

- "Making mistakes is fundamental to learning; they make me aware of my own shortcomings." 67% of females and 100% of males supported this category.
- "Fractions are difficult." 50% of females, but only 8% of males, supported this category.

The reasons for the differing levels of support for these two categories are unclear. Perhaps more interesting is the fact that so many categories showed no significant difference in support levels for females and males. It is conceivable that females and males differ little when it comes to learning algebra as an adult.

Category Support and Ethnicity

The sample was divided into the ethnic groups shown in Table 3. Figure 33 illustrates

Table 3. Sample by Ethnic Group

Ethnicity	Number of Participants
Asian	2
Black	5
Latino	3
White	15

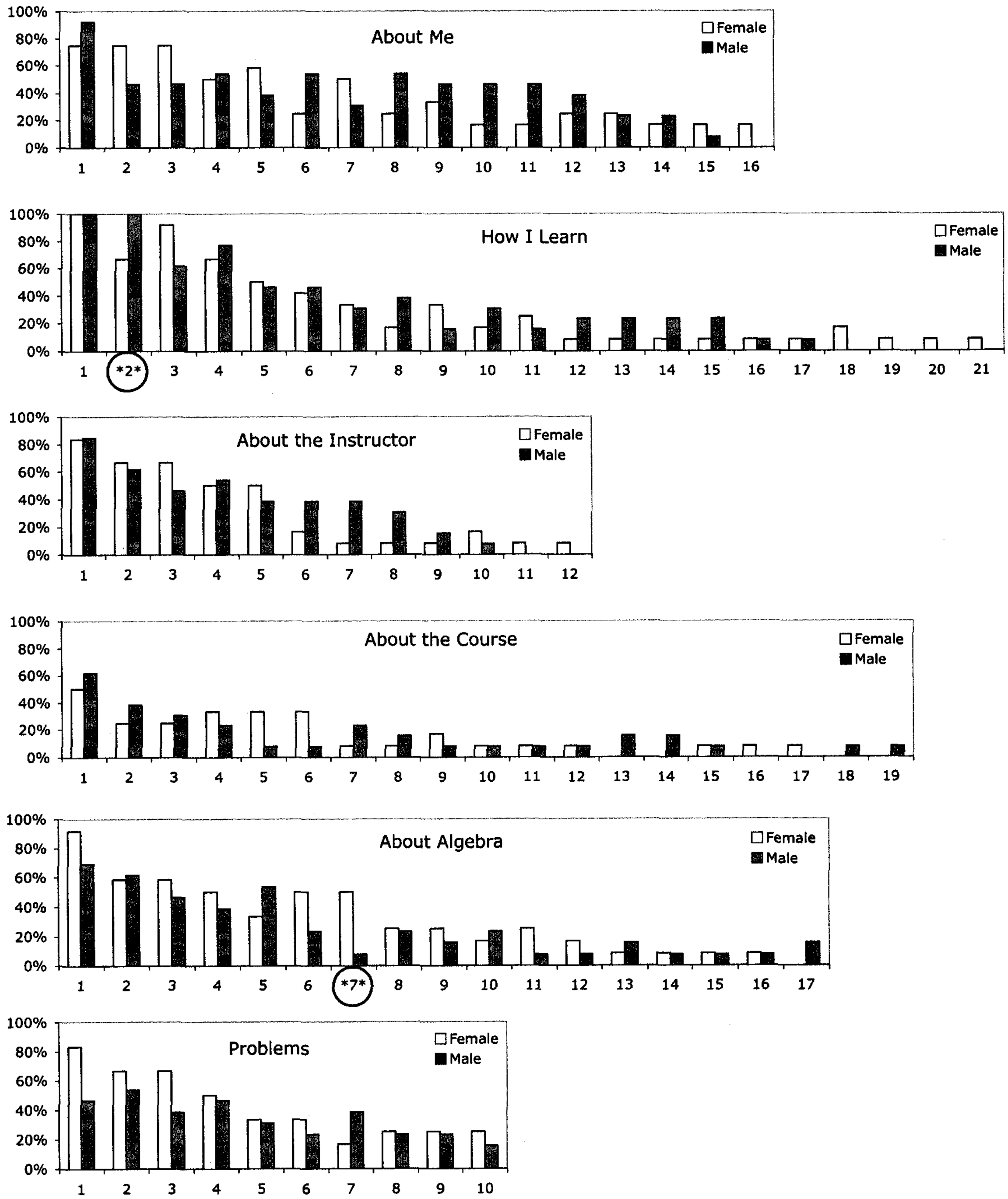

Figure 32. Percentage of each gender supporting each category. Circled categories are those for which levels of support differed significantly between genders.

the percentages of various ethnicities at Mesa College (outer ring) and in the sample (inner ring). There is loose agreement between the population and the sample percentages.

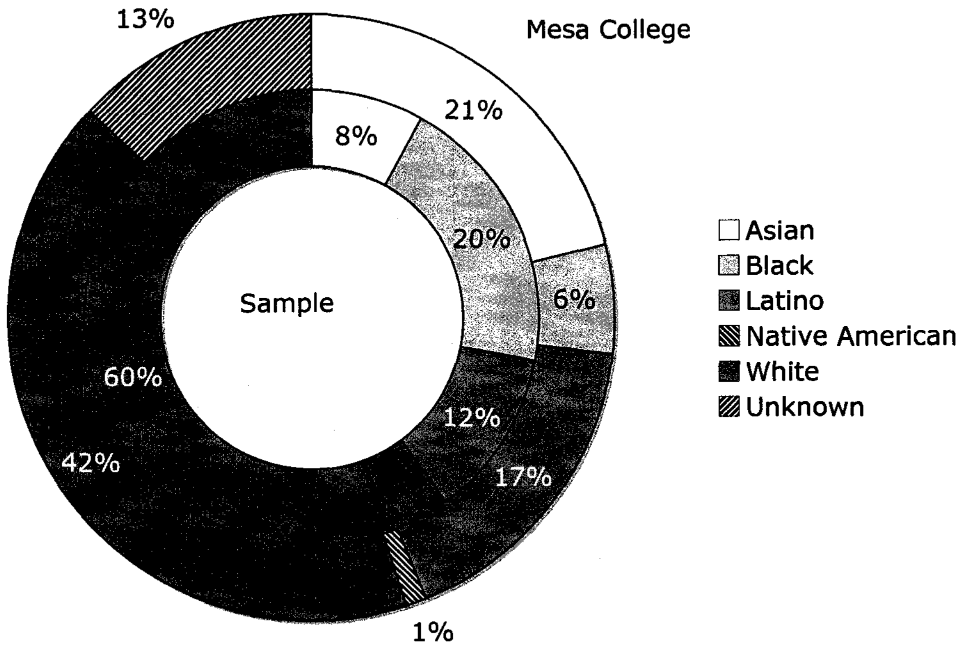

Figure 33. Breakdown by ethnicity of the population of Mesa College students (outer ring) and of the sample (inner ring). The sample is somewhat representative of the population for ethnicity.

Figure 34 shows the percentage of participants for each ethnic group who supported each category. Subcharts in the figure correspond to the supercategories named in the subcharts' titles. Kruskal-Wallis analysis of this data revealed only three categories with significant differences among support levels, possibly due to the low number of participants within each of the ethnic groups. The three categories, all from the **How I Learn** supercategory, were the following.

- "Making mistakes is fundamental to learning; they make me aware of my own shortcomings." 100% of Asians, 40% of Blacks, 100% of Latinos, and 93% of Whites supported this category.
- "When I solve problems myself at the chalkboard, I am more likely to remember the material; it keeps my interest." 50% of Asians, 20% of Blacks, 100% of Latinos, and 7% of Whites supported this category.
- "I don't like to use technology when I learn." 50% of Asians, 0% of Blacks, 33% of Latinos, and 0% of Whites supported this category.

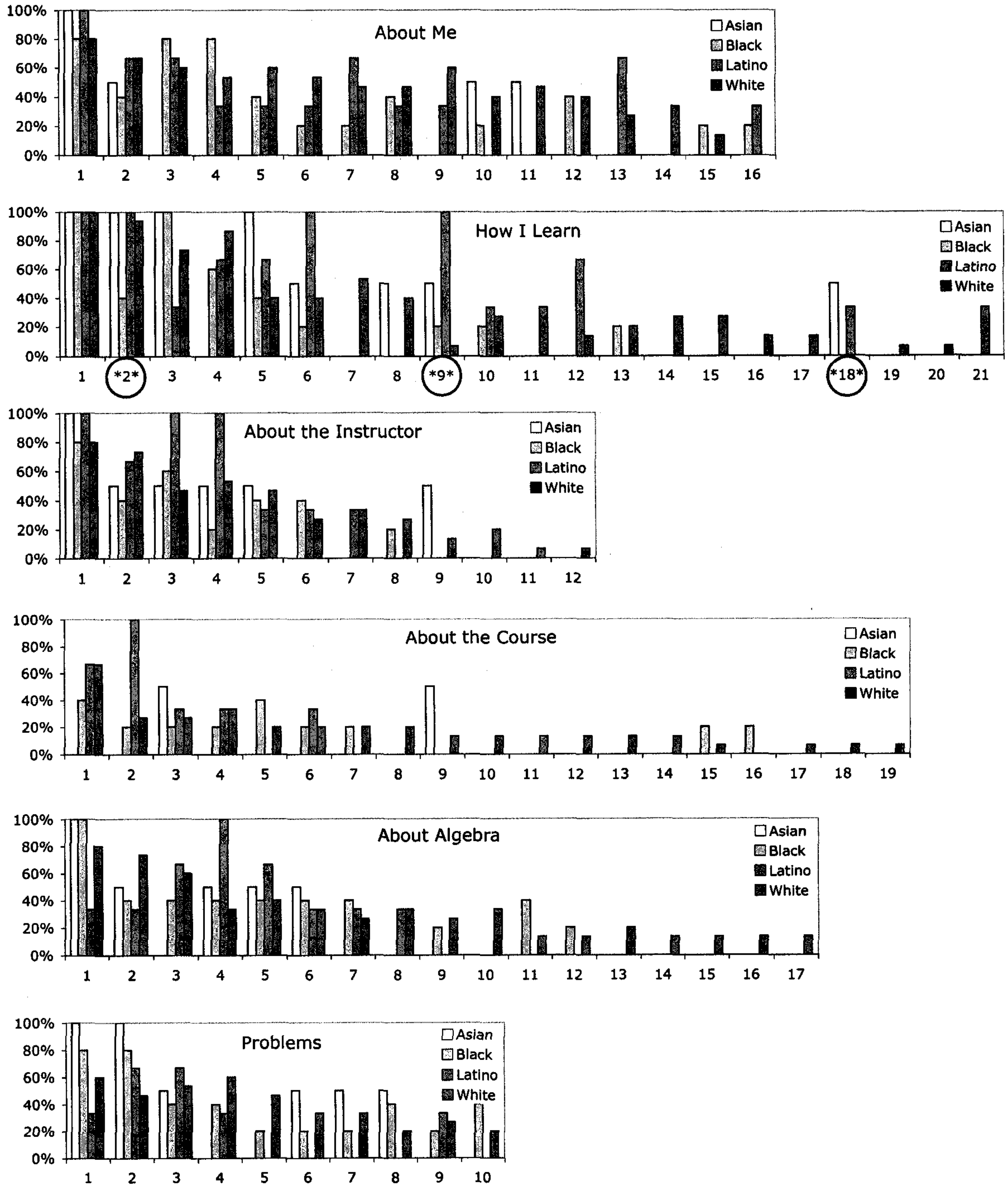

Figure 34. Percentage of each ethnic group supporting each category. Circled categories are those for which levels of support differed significantly among ethnic groups.

The reasons for the differing levels of support for these three categories are unclear. Since each of these categories determines a distinct instructional approach, further research into ethnicity-based instructional preferences seems warranted.

Category Support and Age

The sample was divided into the age groups shown in Table 4. Figure 35 illustrates the percentages of various age groups at Mesa College (outer ring) and in the sample (inner ring). There is relatively close agreement between the population and the sample percentages.

Table 4. Sample by Age Group

Age	Number of Participants
18–25	12
26–35	8
36–48	5

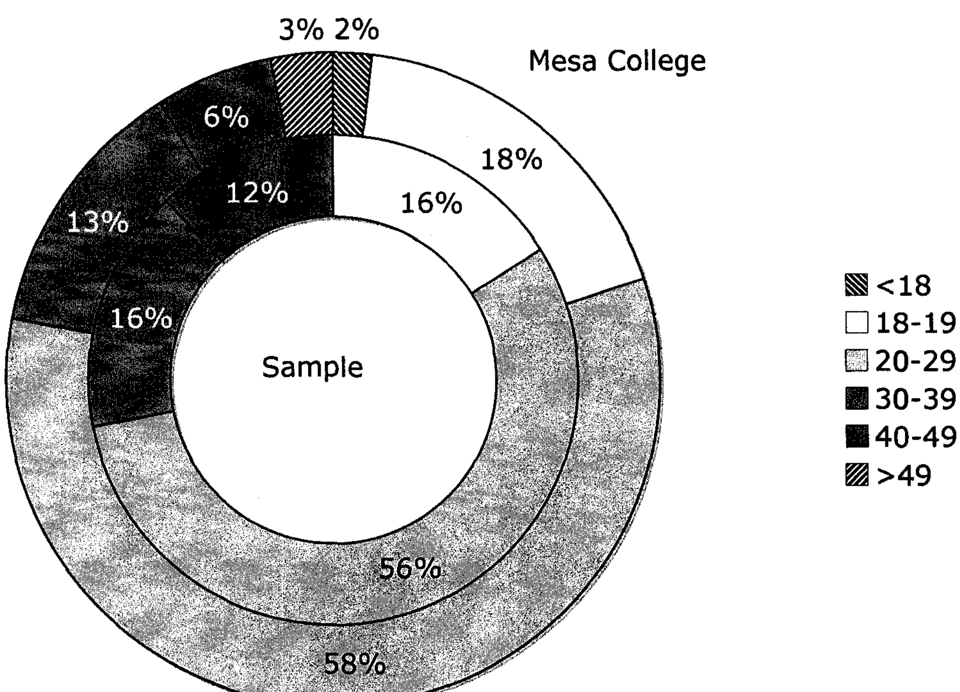

Figure 35. Breakdown by age group of the population of Mesa College students (outer ring) and of the sample (inner ring). The sample is fairly representative of the population for age group.

Figure 36 shows the percentage of participants for each age group who supported each category. Subcharts in the figure correspond to the supercategories named in the subcharts' titles. Kruskal-Wallis analysis of this data revealed five categories with significant differences among support levels. The five categories, one from each supercategory except **Problems**, were the following.

- "I am responsible for my own learning; I just do it." 25% of 18- to 25-year-olds, 63% of 26- to 35-year-olds, and 100% of 36- to 48-year-olds supported this category.
- "I write in my notes everything the instructor puts on the board, because it helps me remember later." 8% of 18- to 25-year-olds, 0% of 26- to 35-year-olds, and 60% of 36- to 48-year-olds supported this category.
- "I like it when instructors give every student personal attention." 0% of 18- to 25-year-olds, 50% of 26- to 35-year-olds, and 40% of 36- to 48-year-olds supported this category.
- "I understand in class, but when I get home, I can't do it." 8% of 18- to 25-year-olds, 13% of 26- to 35-year-olds, and 60% of 36- to 48-year-olds supported this category.
- "Math is fun and exciting when I understand it." 67% of 18- to 25-year-olds, 63% of 26- to 35-year-olds, and 0% of 36- to 48-year-olds supported this category.

The reasons for the differing levels of support for these five categories are unclear, although the increased level of personal responsibility indicated by the first category in the list above may simply be due to increased maturity, i.e., older students may have a more internal locus of control. It is a bit surprising that younger students seem to enjoy math more than older students. Perhaps older students have had less success learning math (hence the reason they are taking algebra at a more advanced age), so math has lost its appeal to them. This, and perhaps academic fossilization, might also explain why they believe they understand in class, but when they get home they feel they cannot do the math.

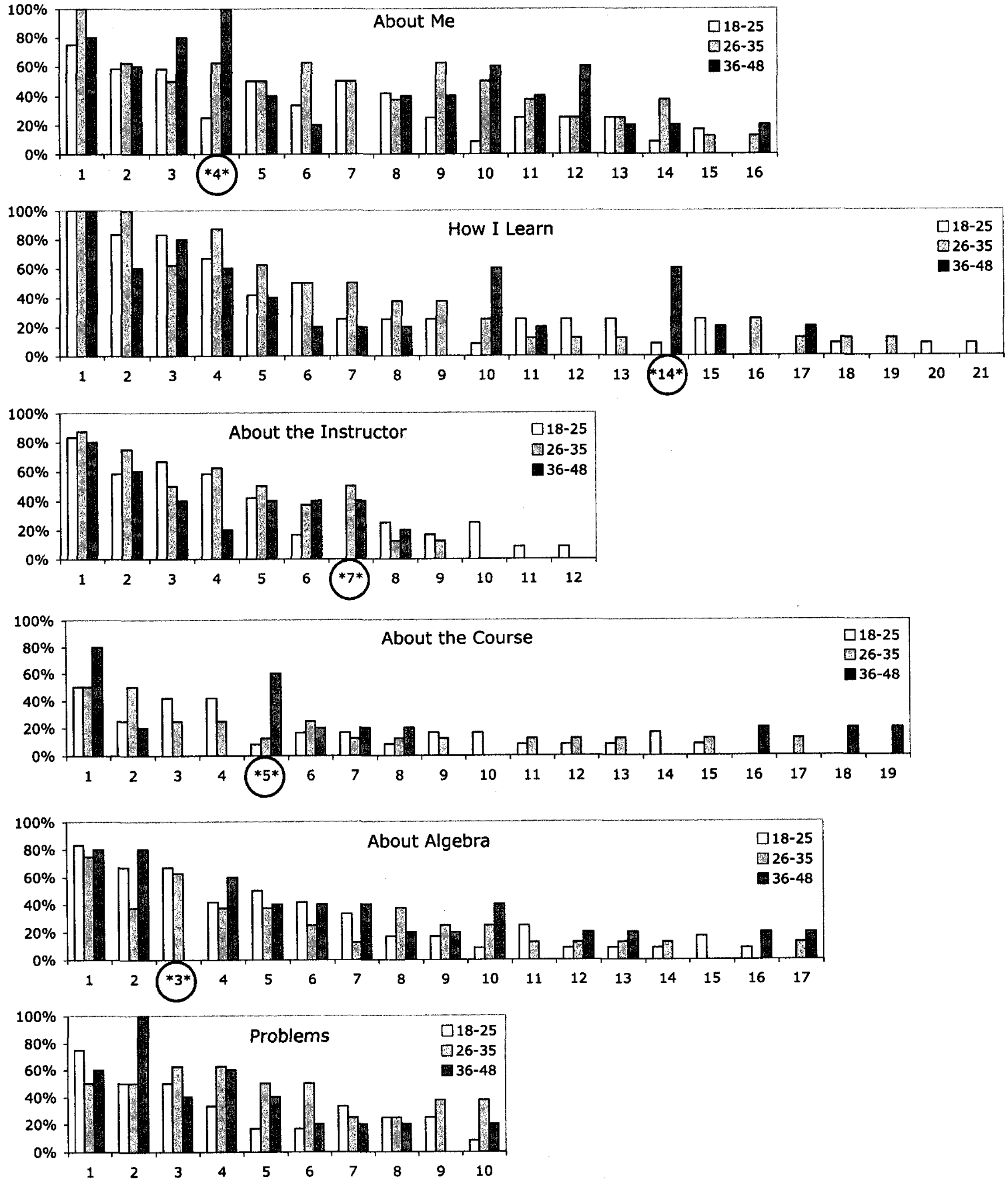

Figure 36. Percentage of each age group supporting each category. Circled categories are those for which levels of support differed significantly among age groups.

CHAPTER 5

CONCLUSIONS

Fluency in algebra is required for employment in many professional fields today, yet it seems to be an elusive goal for many adult students. While a great deal of research has been carried out on child algebra learners and general adult learning, not many studies have focused on the adult algebra learner. Of studies that have been carried out on adult algebra learning, most have been quantitative in nature. Furthermore, these studies invariably make the tacit assumption that most or all factors influencing success in algebra learning lie within the control of the adult student. This study differs from previous studies in its qualitative, exploratory nature and its explicit assumption that success factors may lie within the domain of control of the student or the instructor or institution. Thus, this study offers a novel point of view on a widespread problem and a novel approach to understanding the problem.

Summary of the Study

Research hypotheses for this study included the following.

1. Certain pedagogical factors at Mesa College have a significant influence on the learning of algebra by many adult students.
2. Many adult students have similar misconceptions when they learn algebra to those held by children who are learning algebra. Specifically, they exhibit the following misconceptions: *expression as procedure*, *letter as label*, and *graph as path*.
3. Adults learn algebra more successfully when general adult learning characteristics guide curriculum development. These characteristics include the five specified by Knowles in his theory of andragogy—self-concept, experience, readiness to learn, time, and orientation to learning—as well as academic fossilization and metacognition.

A grounded theory-based methodology was employed to collect and analyze data from 25 adult students taking algebra at Mesa College. Participants were interviewed for about an hour each during the summer and fall of 2005. During these clinical interviews, participants were asked to describe what makes learning algebra difficult. The constant comparison method was employed to tease out 95 categories of responses organized into six supercategories. In the spring of 2006, a survey derived from the response categories was administered to 24 adult students at Mesa College in order to triangulate data analysis results. Due to overwhelming participant and triangulator support, the category "If I can apply it to real life, it is easier to learn and remember" was identified as the core category of the study.

Through an integration process, the core category was connected to all the other categories via the supercategories.

Data were found to confirm the study's research hypotheses.

1. Certain pedagogical factors do indeed influence adult algebra learning, including instructional style and policies, course activities, learning aids, and course pacing. Unanticipated non-pedagogical factors also emerged from the data. For example, adults have a strong sense of personal responsibility. They know they should be persistent in their learning and not give up.
2. When learning algebra, adults do tend to hold some of the misconceptions at rates similar to those of children. These include the *letter as label* and *graph as path* misconceptions, but not the *expression as procedure* misconception. Why adults hold the latter misconception at much lower rates than children is a topic for future research.
3. Adults seem to learn algebra more successfully when general adult learning characteristics guide curriculum development, including those specified by Knowles in his theory of andragogy (self-concept, experience, readiness to learn, time, and orientation), as well as academic fossilization and metacognition. Additional adult learning characteristics also emerged from the data, including concerns over limited memory and a need for feedback.

Strengths and Limitations

This study has several strengths. The unstructured clinical interview process permitted real-time adjustment of interview direction, which allowed me to pursue interesting ideas, investigate promising leads, and test mini-hypotheses with participants. The tokens generated by participants were sincere and revealing, having been freely produced with no requirement that they fulfill any predefined conditions. Grounded theory methodology, with its ability to yield a model based on substantive evidence, is well suited for exploratory studies such as this one for which no prior model exists.

The study also has limitations. I was not able to re-interview the anonymous participants in order to perform member checking; however, I was able to overcome this in part through ongoing triangulation during interviews, i.e., by discussing emerging concepts and categories with later participants during the final portion of their interviews.

The study is limited in the sense that it ignores certain characteristics of participants that may significantly impact their ability to learn algebra. These characteristics may include previous performance in math courses, socioeconomic status, amount of time available for study, previous math courses taken, fluency in English, etc.

Another limitation of this study is inherent to grounded theory. The personal biases, past experience, and knowledge of the grounded theory researcher all have an effect on category formation. Two different researchers using the same set of data might form very distinct categories. To verify that the effect of this limitation was minor, I administered a triangulation survey comprised of the final categories to a set of individuals (triangulators) who were similar to the interviewed participants of the study. The triangulators' agreement with final categories was extensive, suggesting that the final categories were reasonable.

During interviews, some participants were more vocal than others, hence the number of tokens generated by each participant varied quite a bit, with standard deviation $s_{\text{generated}} = 15.4$. To normalize the weighting of each participant's data in the analysis, I based findings on the notion of *support* of a category, i.e., the generation of one or more tokens belonging to a category, for which the standard deviation was only $s_{\text{supported}} = 5.7$.

As in most qualitative studies, the sample size for this study was quite small ($n = 25$); however, I used several measures to ensure that category saturation was achieved by the time I analyzed the final participant's data.

The sample of this study was self selected, hence it may not be highly representative of the population. Furthermore, the sample was obtained from a single institution and may not represent larger populations or populations at other locations. Nevertheless, the sample matched the population of Mesa College students well on gender and age and fairly well on ethnicity. Also, few significant differences of category support were found among distinct genders, ethnicities, and age groups. These latter facts add robustness to the study and increase its probability of being generalizable. In order to truly determine the generalizability of this study, the study should be replicated at other sites. Similar explorations in other areas and with other populations should be carried out and results should be compared to those obtained from this study.

Implications for Future Research

This study indicates the potential existence of a small number of significant learning differences based in student demographics. There were few gender-based differences, but ethnicity-based and age-based differences do appear possible. Further research into ethnicity-based and age-based learning differences is needed.

Some participants in this study appeared to exhibit common misconceptions found among children who are learning algebra; however, it is possible that these were not misconceptions but merely the participants' random attempts at solving problems they did not know how to solve.[1] To discover whether adult algebra learners do tend to exhibit certain misconceptions, further research is needed, including member checking and extensive analysis of written problem-solving work.

[1] This could be true for child algebra learners also.

According to the core category of this study, "If I can apply it to real life, it is easier to learn and remember," virtually all adult algebra learners want algebra to be filled with applications that they can relate to their own lives; nevertheless, about half of them do not want to be tested on applications, rather only on technical skills. It may be that students actually desire algebra to be relevant to their lives, but they do not want to deal with the applications of algebra. To tease this out, further analysis is needed of student comments related to the application and relevance of algebra to real life. Member checking would also help determine this. If it is indeed true that adult algebra learners want to learn real life applications of algebra, then it may be important for institutions to provide in-service training or industry-shadowing opportunities for algebra instructors to enable them to incorporate real-life applications in their courses. Another option would be for institutions to change their hiring policies by requiring job applicants to have prior industrial experience.

Recommendations for Instructors

Adult algebra learners believe that instructors' approaches and attitudes have a large influence on their learning. They feel a personal connection with the subject, but they find it somewhat overwhelming due to the large amount of information it contains. They prefer to learn the portions of the subject that are relevant to their own situations, and they believe that if the subject can be applied to their own lives, they will have an easier time learning and remembering it. The findings of this study suggest many ideas and techniques that may help algebra instructors facilitate learning for their adult students. Several of the most salient of these ideas and techniques are listed below.

- Enable students to find out about their teaching style, course policies, and expectations and how the course will be taught even before class registration.
- Be flexible with policies and teaching methods and be patient with students.
- Make the subject practical and relate it to other disciplines.
- Be organized, clear, and dynamic in speech and writing.
- Allow students to make mistakes in an emotionally safe environment.
- Assign homework to be handed in on a regular basis.
- Evaluate students often and give them regular and timely feedback.
- Keep an eye out for misconceptions such as the *letter as label* misconception and the *graph as path* misconception.
- Be aware of adult students' tendency towards academic fossilization.
- Teach metacognition explicitly.
- Insist that students strive for understanding. Adult algebra students know that reciting formulas and mimicking steps is not the same as knowing and understanding math.

This study may benefit the population of adult algebra learners by imparting awareness to faculty regarding the difficulties encountered by these learners. The more that faculty know about how adults learn algebra and where they encounter difficulties, the better their position to develop and implement pedagogy that addresses those difficulties and takes advantage of adult learning characteristics. If educators can raise the effectiveness of algebra instruction in community colleges, they may be able to reduce the drain of resources caused by students having to take algebra two or three times before they pass. Moreover, if students learn algebra effectively, they will be more likely to excel in advanced courses.

Like much qualitative research, the present study may not be widely generalizable; nevertheless, even if its conclusions can only be extended to Mesa College, it will still serve a large population of faculty and adult learners. If Mesa College algebra instructors take the time to hear what 25 anonymous students have said to them, they could potentially improve instruction for as many as 1500 adult algebra learners every semester.

REFERENCES

Alibali, M. W. (1999). How children change their minds: Strategy change can be gradual or abrupt. *Developmental Psychology, 35*, 127–145.

Amerom, B. A. V. (2003). Focusing on informal strategies when linking arithmetic to early algebra. *Educational Studies in Mathematics, 54*, 63–75.

Ausubel, D. P. (1968). *Educational psychology: A cognitive view.* New York: Holt, Rinehart & Winston.

Babchuk, W. A. (1997). *The rediscovery of grounded theory: Strategies for qualitative research in adult education.* Unpublished doctoral dissertation, University of Nebraska-Lincoln, Lincoln, NB.

Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equals sign. *Mathematics Teaching, 92*, 13–15.

Blumer, H. (1969). *Symbolic interactionism: Perspective and method.* Englewood Cliffs, NJ: Prentice Hall.

Booth, L. R. (1981). Child methods in secondary mathematics. *Educational Studies in Mathematics, 12*(1), 29–41.

Booth, L. R. (1984). *Algebra: Children's strategies and errors.* Windsor, England: NFER-Nelson.

Booth, L. R. (1988). Children's difficulties in beginning algebra. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12: 1988 yearbook* (pp. 20–32). Reston, VA: National Council of Teachers of Mathematics.

Borko, H., & Shavelson, R. (1990). Teacher decision making. In B. F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp. 311–346). Hillsdale, NJ: Erlbaum.

Borkowski, J. G., Carr, M., Rellinger, E., & Pressley, M. (1990). Self-regulated cognition: Interdependence of metacognition, attributions, and self-esteem. In B. F. Jones (Ed.), *Dimensions of thinking: Review of research* (pp. 53–92). Hillsdale, NJ: Erlbaum.

Bowers, J., & Doerr, H. M. (2001). An analysis of prospective teachers' dual roles in understanding the mathematics of change: Eliciting growth with technology. *Journal of Mathematics Teacher Education, 4*, 115–137.

Braswell, J. S., Lutkus, A. D., Grigg, W. S., Santapau, S. L., Tay-Lim, B. S.-H., & Johnson, M. S. (2001). *The nation's report card: Mathematics 2000.* Washington, DC: National Center for Education Statistics.

Carraher, D., Schliemann, A. D., & Brizuela, B. M. (2000, October). *Early algebra, early arithmetic: Treating operations as functions.* (Plenary address. XXII Meeting of the Psychology of Mathematics Education, North American Chapter, Tucson, AZ.)

Chalouh, L., & Herscovics, N. (1984). From letter representing a hidden quantity to letter representing an unknown quantity. In J. Moser (Ed.), *Proceedings of the 6th annual meeting of the psychology of mathematics education, North American group* (pp. 71–76). Madison, WI: University of Wisconsin.

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13*, 16–30.

Clement, J., Lochhead, J., & Monk, G. S. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly, 88*(4), 286–290.

Clement, L. L. (2001, December). What do students really know about functions? *Mathematics Teacher, 94*(9), 745–748.

Cohen, R. (2006). *What is the value of algebra?* (Retrieved on 10 June 2006 from http://www.washingtonpost.com/wp-dyn/content/blog/2006/02/15/BL2006021501989.html.)

Collis, K. F. (1975). *A study of concrete and formal operations in school mathematics: A Piagetian viewpoint.* Melbourne: Australian Council for Educational Research.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions.* Thousand Oaks, CA: Sage.

Davis, R. B. (1975). Cognitive processes involved in solving simple algebraic equations. *Journal of Children's Mathematical Behaviour, 1*(3), 7–35.

Dey, I. (1993). *Qualitative data analysis: A user-friendly guide for social scientists.* London: Routledge.

Dick, B. (2002). *Grounded theory: A thumbnail sketch.* (Retrieved on 30 June 2004 from http://scu.edu.au/schools/gcm/ar/arp/grounded.html.)

Doyle, W. (1986). Classroom organization and management. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 392–431). New York, NY: Macmillan Publishing Company.

Duell, O. K. (1986). Metacognitive skills. In G. Phye & T. Andre (Eds.), *Cognitive instructional psychology: Components of classroom learning* (pp. 205–242). Orlando, FL: Academic Press.

Dugdale, S. (1993). Functions and graphs—perspectives on student thinking. In T. A. Romberg, E. Fennema, & T. P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 101–130). Hillsdale, NJ: Lawrence Erlbaum Associates.

Eisenbud, D. (2004). *Why Johnny can't add: Improve math class, save jobs for kids.* (Retrieved on 10 August 2004 from http://www.sfgate.com/cgi-bin/article.cgi?file=/chronicle/archive/2004/03/12/EDGAT5J2I21.DTL.)

Ekenstam, A. A., & Nilsson, M. (1979). A new approach to the assessment of children's mathematical competence. *Educational Studies in Mathematics, 10*, 41–66.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist, 34*, 906–911.

Gardner, H. (1985). *The mind's new science: A history of the cognitive revolution.* New York: Basic Books.

Gertsen, M. (2004). *Funding for math education research.* (Retrieved on 24 December 2004 from personal.bgsu.edu/~schuetj/Funding%20for%20Math.ppt.)

Ginsburg, H. P. (1975). Young children's informal knowledge of mathematics. *Journal of Children's Mathematical Behaviour, 1*(3), 63–156.

Ginsburg, H. P. (1977). *Children's arithmetic: The learning process.* New York: Van Nostrand.

Ginsburg, H. P. (1997). *Entering the child's mind: The clinical interview in psychological research and practice.* Cambridge: Cambridge University Press.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* Chicago: Aldine de Gruyter.

Gouvêa, F. Q. (2006, May/June). Parents aren't worried about math and science. *Focus, 26*(5), 16.

Greenberg, R. L. (1991). *Factors affecting achievement in algebra at a community college.* (Paper presented at the 14th Annual Meeting of the Southwest Educational Research Association, San Antonio, TX, 24–26 January 1991.)

Harrison, W.-J., & Teegarden, T. (2004). *Pathways through algebra: Definition.* (Retrieved on 30 August 2004 from http://www.algebrapathways.org/success.asp.)

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524–550.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1–27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Johnson, J., Arumi, A. M., Ott, A., & Remaley, M. H. (2006). *Are parents and students ready for more math and science?* New York, NY. (Retrieved on 10 June 2006 from http://www.publicagenda.org/research/pdfs/rc0601.pdf.)

Karsenty, R. (2002). What do adults remember from their high school mathematics? the case of linear functions. *Educational Studies in Mathematics, 51*, 117–144.

Kieran, C. (1979). Children's operational thinking within the context of bracketing and the order of operations. In D. Tall (Ed.), *Proceedings of the 3rd international conference for the psychology of mathematics education* (pp. 128–132). Warwick, England: University of Warwick.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*(3), 317–326.

Kieran, C. (1988). Two different approaches among algebra learners. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12: 1988 yearbook* (pp. 91–96). Reston, VA: National Council of Teachers of Mathematics.

Knowles, M. S. (1980). *The modern practice of adult education: From pedagogy to andragogy.* Englewood Cliffs, NJ: Prentice Hall.

Knowles, M. S. (1984). *The adult learner: A neglected species* (3rd ed.). Houston, TX: Gulf Publishing.

Kra, I., & Robinson, W. G. (2006, November). Mathematics teaching and American competitiveness. *Focus, 26*(8), 18–19.

Kuchemann, D. E. (1981). Algebra. In K. Hart (Ed.), *Children's understanding of mathematics* (pp. 11–16). London: Murray.

Lapp, D. A., & Cyrus, V. F. (2000, September). Using data-collection devices to enhance students' understanding. *Mathematics Teacher, 93*(6), 504–510.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990, Spring). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1–64.

Lerch, C. (2002). *The unfossilization of concepts in adult learning.* (Paper presented at the 22nd Lilly Conference on College Teaching, Miami University, Oxford, OH, 21–24 November 2002.)

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Thousand Oaks, CA: Sage.

Lochhead, J., Eylon, B.-S., Ikeda, H., & Kishor, N. (1985). *Representation of mathematical relationships in four countries.* (Paper presented at the Annual Meeting of the Mathematical Association of America, Anaheim, CA, 7–13 January 1985.)

Lochhead, J., & Mestre, J. P. (1988). From words to algebra: Mending misconceptions. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12: 1988 yearbook* (pp. 127–135). Reston, VA: National Council of Teachers of Mathematics.

Lutzer, D. J., Maxwell, J. W., & Rodi, S. B. (2006). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2000 CBMS survey.* Providence, RI: American Mathematical Society.

MacGregor, M., & Stacey, K. (1994). Progress in learning algebra: Temporary and persistent difficulties. In G. Bell, B. Wright, N. Leeson, & J. Geake (Eds.), *Challenges in mathematics education: Constraints on construction* (Vol. 2, pp. 303–410). Lismore, Australia: Mathematics Education Research Group of Australasia.

Markovits, Z., Eylon, B. S., & Bruckheimer, M. (1983). Functions: Linearity unconstrained. In R. Hershkowitz (Ed.), *Proceedings of the 7th conference of the international group for the psychology of mathematics education* (pp. 271–277). Rehovot, Israel: Weizmann Institute, Department of Science Teaching.

Markovits, Z., Eylon, B. S., & Bruckheimer, M. (1988). Difficulties students have with the function concept. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12: 1988 yearbook* (pp. 43–60). Reston, VA: National Council of Teachers of Mathematics.

Matz, M. (1980). Towards a computational theory of algebraic competence. *Journal of Children's Mathematical Behaviour, 3*(1), 93–166.

McLoughlin, J. A., & Lewis, R. B. (1994). *Assessing special students* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

McNeil, N. M., & Alibali, M. W. (2000). Learning mathematics from procedural instruction: Externally imposed goals influence what is learned. *Journal of Educational Psychology, 92*, 734–744.

McNeil, N. M., & Alibali, M. W. (2005a). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition and Development, 6*(2), 285–306.

McNeil, N. M., & Alibali, M. W. (2005b). Why won't you change your mind? knowledge of operational patterns hinders learning and performance on equations. *Child Development, 76*(4), 883–899.

Merriam, S. B. (2001). Andragogy and self-directed learning: Pillars of adult learning theory. *New Directions for Adult and Continuing Education, 89*, 3–13.

Mestre, J. P. (1987). Why should mathematics and science teachers be interested in cognitive research findings? *Academic Connections*, 3–5, 8–11.

Mestre, J. P., & Gerace, W. J. (1986). The interplay of linguistic factors in mathematical translation tasks. *Focus on Learning Problems in Mathematics, 8*, 59–72.

Mestre, J. P., Gerace, W. J., & Lochhead, J. (1982). The interdependence of language and translational math skills among bilingual Hispanic engineering students. *Journal of Research in Science Teaching, 19*, 399–410.

Mestre, J. P., & Lochhead, J. (1983). The variable-reversal error among five cultural groups. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the 5th annual meeting of the North American chapter of the international group for the psychology of mathematics education* (pp. 180–188). Montreal: International group for the psychology of mathematics education.

Moses, R. P. (1995). Algebra, the new civil right. In C. B. Lacampagne, W. Blair, & J. Kaput (Eds.), *The algebra initiative colloquium* (Vol. 2, pp. 53–67). Washington, DC: U.S. Department of Education.

Moses, R. P., Kamii, M., Swap, S. M., & Howard, J. (1989). The Algebra Project: Organizing in the spirit of Ella. *Harvard Educational Review, 59*(4), 423–443.

Mullis, I. V. S., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1991). *The state of mathematics achievement: NAEP's 1990 assessment of the nation and the trial assessment of the States.* Washington, DC: U. S. Department of Education.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 international mathematics report.* Chestnut Hill, MA: International Association for the Evaluation of Educational Achievement.

Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education, 31*(2), 168–190.

National Commission on Excellence in Education. (1983, April). *A nation at risk: The imperative for educational reform.* Washington, DC.

NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Norton, S. J., & Cooper, T. J. (2001, August). Students' perceptions of the importance of closure in arithmetic: Implications for algebra. In A. Rogerson (Ed.), *New ideas in mathematics education* (pp. 198–202). Brisbane, Australia: Centre for Mathematics and Science Education.

Ocken, S. (2004). *Why students fail calculus.* (Retrieved on 11 August 2004 from http://www.nychold.com/ocken-calculus01.pdf.)

O'Sullivan, C. Y., Reese, C. M., & Mazzeo, J. (1997). *NAEP 1996 science report card for the nation and the states.* Washington, DC: National Center for Education Statistics.

Paris, S. G., & Winograd, P. (1990). How metacognition can promote academic learning and instruction. In B. F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp. 15–52). Hillsdale, NJ: Erlbaum.

Patton, M. Q. (2001). *Qualitative evaluation and research methods* (3rd ed.). Thousand Oaks, CA: Sage.

Petitto, A. (1979). The role of formal and informal thinking in doing algebra. *Journal of Children's Mathematical Behaviour, 2*(2), 69–88.

Poochigian, C. (2000). *California senate bill SB 1354: Algebra instruction.*

Poochigian, C. (2004). *Nearly 13,000 students granted waivers from basic algebra requirement.* (Retrieved on 1 September 2004 from http://republican.sen.ca.gov/opeds/14/oped2225.asp.)

Preis, C., & Biggs, B. T. (2001, April/May). Can instructors help learners overcome math anxiety? *American Technical Education Association Journal, 28*(4), 6–10.

Pressley, M., Borkowski, J. G., & Schneider, W. (1987). Good strategy users coordinate metacognition, strategy use and knowledge. In R. Basta & G. Whitehurst (Eds.), *Annals of child development* (Vol. 5, pp. 89–129). Greenwich, CT: JAI Press.

Rachal, J. R. (2002). Andragogy's detectives: A critique of the present and a proposal for the future. *Adult Education Quarterly, 52*(3), 210–227.

Resnick, L. B. (1982, April). Mathematics and science learning: A new conception. *Science, 20*(4), 477–478.

Rosnick, P. (1981). Some misconceptions concerning the concept of variable. *Mathematics Teacher, 74*(6), 418–420.

Rosnick, P., & Clement, J. (1980). Learning without understanding: The effect of tutoring strategies on algebra misconceptions. *Journal of Mathematical Behavior, 3*(1), 3–27.

Russell, B. (1918). *Mysticism and logic.* London: Longmans Green.

San Diego Community College District Institutional Research and Planning. (2004). *Student outcomes.* (Retrieved on 1 September 2004 from http://www.sdccd.net/research/archives_student_outcomes/MathPlacement_success.PDF.)

San Diego Community College District Institutional Research and Planning. (2006). *SDCCD student profile, spring 2006 (census).* (Retrieved on 14 June 2006 from http://research.sdccd.edu/enrollment/Profile_SP06_c.htm.)

Schoenfeld, A. (1985). *Mathematical problem solving.* New York: Academic Press.

Schoenfeld, A. (1987). *Cognitive science and mathematics education.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22,* 1–36.

Shulman, L. S. (1984). Disciplines of inquiry in education: An overview. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (pp. 3–20). Washington, DC: National Academy Press.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*(4), 476–522.

Small, D. (2006). *An urgent call to improve traditional college algebra programs.* (Retrieved on 10 December 2006 from http://www.maa.org/t_and_l/urgent_call.html.)

Smith, F. (2002). *The glass wall: Why mathematics can seem difficult.* New York: Columbia University Teachers College Press.

Soloway, E., Lochhead, J., & Clement, J. (1982). Does computer programming enhance problem solving ability? Some positive evidence on algebra word problems. In R. J. Seidel, R. E. Anderson, & B. Hunter (Eds.), *Computer literacy issues and directions for 1985* (pp. 171–215). New York: Academic Press.

Stacey, K., & MacGregor, M. (1994, August). Algebraic sums and products: Students' concepts and symbolism. In J. P. d. Ponte & J. F. Matos (Eds.), *Proceedings of the eighteenth international conference for the psychology of mathematics education* (pp. 289–296). Melbourne, Australia: University of Melbourne.

Steen, L. A. (1989). Mathematics for a new century. *Notices of the American Mathematical Society, 36*(2), 133–138.

Strauss, A. L. (1987). *Qualitative analysis for social scientists.* Cambridge: Cambridge University Press.

Strauss, A. L., & Corbin, J. M. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 273–285). Thousand Oaks, CA: Sage.

Tall, D., Gray, E., Ali, M. B., Crowley, L., DeMarois, P., McGowen, M., et al. (2000). Symbols and the bifurcation between procedural and conceptual thinking. *Canadian*

Journal of Mathematics, Science, and Technology Education, 1(1), 81–104.

Tall, D., & Razali, M. R. (1993). Diagnosing students' difficulties in learning mathematics. *International Journal of Mathematics Education in Science and Technology, 24*(2), 209–227.

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics, 22*(2), 125–147.

Teachey, A. L. (2003). *Investigations in conceptual understanding of polynomial functions and the impact of mathematical belief systems on achievement in an accelerated summer program for gifted students.* Unpublished doctoral dissertation, North Carolina State University, Raleigh, NC.

Thompson, A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A. (1994). Calculational and conceptual orientations in teaching mathematics. In A. Coxford (Ed.), *1994 yearbook of the NCTM* (pp. 79–92). Reston, VA: NCTM.

Toubassi, E. (1999). Department goals and assessment. In B. Gold, S. Z. Keith, & W. A. Marion (Eds.), *Assessment practices in undergraduate mathematics* (pp. 61–63). Washington, DC: Mathematical Association of America.

United States Congress. (2002). *Public law 107-110: No child left behind act of 2001.*

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12: 1988 yearbook* (pp. 8–19). Reston, VA: National Council of Teachers of Mathematics.

Vygotsky, L. S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.

Wagner, S. (1977). *Conservation of equation, conservation of function, and their relationship to formal operational thinking.* Unpublished doctoral dissertation, New York University, New York, NY.

Waycaster, P. (2001). Factors impacting success in community college developmental mathematics courses and subsequent courses. *Community College Journal of Research and Practice, 25*(5/6), 403–416.

Zmeyov, S. I. (1998). Andragogy: Origins, developments and trends. *International Review of Education, 44*(1), 103–108.

APPENDIX A

RECRUITMENT ANNOUNCEMENT

My name is Michael Reese. I am a math professor at Mesa College. As part of my doctoral studies at San Diego State University, I am conducting research to find out what is difficult about learning algebra. I am asking Mesa students in Math 95 or Math 96 who are at least 18 years old to let me interview them for about one hour on the Mesa College campus.

Your participation in this study is completely anonymous and voluntary. Your decision whether or not to participate will not affect your grade for Math 95 or Math 96.

During the interview, I will ask questions about difficulties you may have with algebra, how you solve algebra problems, and how you feel you can best learn algebra. If you are uncomfortable answering some questions or expressing your opinions, you do not have to respond to those questions or give opinions.

You will be paid $10 for participating in this study. There are no costs to you for participating.

If you would like to participate in my study, please contact me at `mreese@sdccd.net` or (619) 388-2382, or you may stop by my Mesa office in H209B.

Thank you.

APPENDIX B

RECRUITMENT FLYER

Participants Wanted for Research Study

What's So Hard About Algebra? A Grounded Theory Study of Adult Algebra Learners

- Are you at least 18 years old?
- Are you a Mesa College student enrolled in Math 95 or Math 96?
- Are you willing to share your thoughts about algebra in a one-hour interview?

If you answer "yes" to these questions, then you may be eligible to participate in a confidential research study on what makes learning algebra difficult.

The study will take place at Mesa College during the summer and fall of 2005. Participants will be paid for their time.

If you wish to participate, please contact the researcher:

Professor Michael Reese
Email: `mreese@sdccd.net`
Phone: (619) 388-2382
Office: H209B

APPENDIX C
INSTITUTIONAL REVIEW BOARD PERMISSIONS

From: Wendy Bracken [SMTP:wbracken@mail.sdsu.edu]
To: mreese@sdccd.net
Cc: jbowers@math.sdsu.edu
Subject: Approval of your protocol
Sent: 11/4/04 10:07 AM **Importance:** Normal

Dear Mr. Reese:

The protocol #126 "What's So Hard About Algebra? A Grounded Theory Study of Adult Algebra Learners" was reviewed and verified as exempt in accordance with SDSU's Assurance and federal requirements pertaining to human subjects protections within the Code of Federal Regulations (45 CFR 46.101(b)). This review is valid through October 11, 2005, and applies to the conditions and procedures described in your protocol. If any changes to your study are planned or you require additional time to complete your project, please notify the IRB office. Additionally, notify the IRB office if your status as an SDSU-affiliate changes while conducting this research study (you are no longer an SDSU faculty member, staff member or student).

Please note: If this research involves the use of existing or secondary data sources, information obtained must be recorded so that subjects cannot be identified, either directly or through identifiers linked to the subjects. If information will be obtained from an individual's medical record, please check with the organization authorized to provide access to these records to determine whether regulations relating to the Health Insurance Portability and Accountability Act (HIPAA) pertain to your research. Likewise, if academic records are accessed, Federal Education Rights and Privacy Act (FERPA) requirements must be respected. Notify the SDSU IRB office if protocol revisions are necessary to comply with HIPAA regulations.

For questions related to this correspondence, please contact the IRB office ((619) 594-6622 or e-mail irb@mail.sdsu.edu). To access IRB review application materials, SDSU's Assurance, the 45 CFR 46, the Belmont Report, and/or any other relevant policies and guidelines related to the involvement of human subjects in research, please visit the IRB web site at http://gra.sdsu.edu/irb.

Graduate Students: This letter may be used to verify approval by the SDSU Institutional Review Board (IRB) for enrollment in Thesis 799A. If you are not presently enrolled in 799A, attach the enclosed copy of this letter to your *Appointment of Thesis/Project Committee* form prior to submitting the completed form to Graduate and Research Affairs/Student Services Division. If you enrolled in 799A using the IRB e-mail notification, please forward the enclosed copy of this final approval letter to the Graduate Division for completion of your record.

Thank you,

Wendy Bracken
Regulatory Compliance Analyst

From: SDSU e-Services WebPortal [SMTP:eservices@mail.sdsu.edu]

To: Michael Reese

Cc:

Subject: Your Report of Progress was reviewed

Sent: 9/28/05 9:30 AM **Importance:** Normal

Protocol Title:	What's So Hard About Algebra? A Grounded Theory Study of Adult Algebra Learners
Protocol Number:	126
Status:	Progress Reported
Principal Investigator:	Michael Reese

Dear Michael Reese:

The SDSU Institutional Review Board approved the project referenced for continuation on 09/28/2005 in compliance with federal regulations. This approval applies to the collection and/or analysis of data collected based on procedures described in your protocol. Approval carries with it the understanding that you will contact the IRB to obtain authorization to implement any proposed changes to the protocol, to document a change in your affiliation with SDSU (student, faculty or staff), and/or to report study completion (recruitment, data collection and analysis).
Your project is subject to continuing review by the IRB. Approximately six weeks before approval expires, we will send you an electronic message as a reminder that you are required to submit a progress report. This progress report must be reviewed and approved by the IRB prior to 10/11/2005. As it is your responsibility to submit this report, please notify the IRB office if you do not receive this reminder message.

For questions related to this correspondence, please contact the IRB office ((619) 594-6622 or irb@mail.sdsu.edu). To access IRB review application materials, SDSU's Assurance, the 45 CFR 46, the Belmont Report, and/or any other relevant policies and guidelines related to the involvement of human subjects in research, please visit the IRB web site.

From: Yvonne Bergland
Sent: Monday, October 18, 2004 11:27 AM
To: Michael Reese
Cc: Dina Miyoshi; Yosh Kawahara; Jaye Van Kirk; ubehave@tns.net ; Caterina Palestini
Subject: IRB's Finding RE: Your Research at Mesa College
Importance: High

Michael,

The Mesa College IRB reviewed the additional materials they requested concerning your study "What's So Hard About Algebra? A Grounded Theory Study of Adult Algebra Learners".

After careful consideration of these materials, committee members are in agreement to approve your study at Mesa College.

If you have any further questions, please feel free to contact either me or members of the committee.

Yvonne
Yvonne Bergland, Ph.D, Dean
Instructional Services and Economic Development
San Diego Mesa College
7250 Mesa College Drive
San Diego, CA 92111
E-mail: yberglan@sdccd.net
Phone: (619) 388-2509

APPENDIX D

CONSENT FORM

Dear Prospective Study Participant

I am a student in the educational technology department at San Diego State University and the University of San Diego and a mathematics professor at Mesa College. I am conducting a study to identify what is difficult about algebra for adult students at Mesa College. I am asking students in Math 95 or Math 96 at Mesa College who are at least 18 years old to allow me to interview them for approximately one hour at a quiet location on the Mesa College campus. The results will be reported in a dissertation that I will complete as a requirement of my graduate program. Professor Janet Bowers of the mathematics department at San Diego State University is supervising my research.

Your participation in this study is completely anonymous and voluntary. Your decision whether or not to participate in this study will not affect your grade for Math 95 or Math 96. All information you provide during the study will be seen only by me. I will record the interview using a digital voice recorder so that I can transcribe it later. I will store the digital voice recorder securely at my home or on my person. Once I have transcribed the interview, I will erase the recording of the interview. I will store the transcribed data securely on my office computer, and once the study is completed, I will erase the transcribed data.

During the interview, I will ask your age and ethnicity. I will ask what you find difficult about algebra. I may ask you to demonstrate or explain how you solve various algebra problems. I may ask you to describe your experiences with or opinions about algebra, learning, and teaching, including how you feel you can best learn algebra. I may take written notes, and I will collect your written problem-solving work. Once the study is completed, I will destroy all written notes and data.

By giving you and other students the opportunity to express yourselves anonymously about your learning experiences, I hope to develop a theory of adult algebra learning so that we may improve instruction at Mesa College.

You may feel uncomfortable answering some of the questions or expressing your opinion about algebra instruction or learning. If any questions make you feel uncomfortable, you do not have to answer them, and you can discontinue participation at any time without penalty.

You will be paid $10 for participating in this study. There are no costs to you for participating.

If you have any questions related to this study, you may ask now or contact me later by email or telephone. You may stop by my Mesa College office to see me in person. If you wish to report problems or concerns regarding this study, you may contact the Institutional Review Board at SDSU (619-594-6622, or irb@mail.sdsu.edu) or at Mesa College (619-388-2509). You may also contact the Office of the Vice President and Provost at USD (619-260-4553).

Thank you for participating in my study.

Michael Reese
Email: mreese@sdccd.net
Telephone: (619) 388-2382
Mesa College office: H209B

APPENDIX E

CLINICAL INTERVIEW GUIDE

Pre-Interview (5 minutes)

1. Give informed consent and discuss
2. Start recording

Interview (45 – 60 minutes)

1. Ask age and ethnicity of participant

2. Explore hypothesis 1 (15 minutes): Adult students at Mesa College face a small set of common conceptual and affective difficulties when they are learning algebra, and these difficulties tend to reduce students' chances of being successful in algebra classes.

    a. *Please describe the difficulties you have with learning algebra and how these difficulties have affected your performance in class.*
    b. *Are there parts of algebra you just don't "get"? Please describe them.*
    c. *What do you like or dislike about algebra? Please comment on both your own learning and teaching that occurs in the classroom.*

3. Explore hypothesis 2 (25 minutes): Adult algebra learners have the same misconceptions that children have when they learn algebra, as well as other conceptual difficulties, not experienced by children, resulting from academic fossilization.

    a. Expression as procedure: *On paper, solve $1 + (x + yz) = 2(x + yz)$ for x. Explain your thinking.*
    b. Letter as label: *Suppose there are six times as many students as professors at Mesa College. Write this relationship mathematically using variables S and P. Explain your thinking.*
    c. Graph as path: *Sketch a graph of speed (y) versus time (x) for a bicycle going up a steep hill and down the other side. Explain your thinking.*

4. Explore hypothesis 3 (20 minutes): When adult learning characteristics are ignored in the classroom, adults find algebra learning more difficult.

    a. *Do you feel a need to be involved in instructional planning and evaluation, i.e., how the course is taught and how you are graded? Explain.*
    b. *Is it important for you to learn from experience, including making mistakes? Explain.*
    c. *Do you think what you are learning should have personal relevance to you? Explain.*
    d. *Do you think learning should concentrate on solving problems rather than merely learning content? Explain.*

5. Stop recording

Post-Interview (5 minutes)

1. Express thanks
2. Compensate $10

APPENDIX F

TUTORING AND DSPS AT MESA COLLEGE

Free tutoring and services for students with physical and learning disabilities are available to students at Mesa College. If you have any questions about these services, you may contact Lori Adrian, Dean of Student Affairs, at (619) 388-2699 or `ladrian@sdccd.net`.

The **Tutoring Center** provides tutoring in most subject areas, with emphasis on math and science. The Center, located in K211, is open Monday through Thursday from 8:30 a.m. to 6:30 p.m. and Friday from 8:30 a.m. to 2:00 p.m. It is available on a walk-in basis or by appointment. Erica Specht (`especht@sdccd.net`) co-coordinates the Center with William Peters (`wpeters@sdccd.net`). You can reach the Center at (619) 388-2898.

An interdisciplinary **Writing Center** also provides one-on-one and small group assistance to students with all aspects of writing across all disciplines. The Writing Center, located in C108, is open during the days and times listed below.

Monday 10:00 a.m. to 1:00 p.m.	Wednesday 10:00 a.m. to 2:00 p.m.
Tuesday 11:00 a.m. to 3:00 p.m.	Thursday 11:00 a.m. to 2:00 p.m.

The Writing Center is available on a walk-in basis or by appointment. The co-coordinators of the Writing Center are Robert Pickford (`rpickfor@sdccd.net`) and David Klowden (`dklowden@sdccd.net`). You can reach the Writing Center at (619) 388-2570.

The **Bridging Skills Lab** offers individualized assistance and skill development in writing, reading and comprehension, ESOL, math, and study techniques. Located in H218, the Lab is open from Monday through Thursday, from 8:30 a.m. to 2:30 p.m. Jeanine Eberhardt (`jeberhar@sdccd.net`) and Carl Luster (`cluster@sdccd.net`) are co-coordinators of the Lab. You can reach the Lab at (619) 388-2869.

The **STAR program** offers individual tutoring in all subject areas by appointment for the entire semester. This program is designed for low income, first generation college students or students with disabilities with an academic need. Students must apply in I300-101. The coordinator of the STAR program is Marichu Magana (mmagana@sdccd.net) at (619) 388-2706.

The **Disabled Students Programs and Services** (DSPS) department is the campus office responsible for providing academic accommodations for students with disabilities, such as test proctoring, sign language interpreting, priority registration, and disability management counseling. You may contact DSPS at (619) 388-2780 or (619) 388-2974 (tty), or you may stop by the DSPS office at H202. A complete orientation to DSPS is available online at `http://www.sdmesa.sdccd.net/~lon/dsps/`.

APPENDIX G

TRIANGULATION SURVEY

The following three pages contain the triangulation survey of 85 Likert items derived from the final categories of the study. The survey was administered to 24 anonymous adult algebra students at Mesa College enrolled in Math 95 (Beginning Algebra) and Math 96 (Intermediate Algebra) during the spring 2006 semester. The instructions on the survey indicated that respondents should mark their level of personal agreement for each item according to the following schedule.

SA: Strongly Agree

A: Agree

D: Disagree

SD: Strongly Disagree

		SA	A	D	SD
1	Past experience aids my current learning; it makes things familiar; you get a practical understanding				
2	I wish I felt more confident about my math abilities				
3	Being well rounded is important to me, and math is part of that				
4	I am responsible for my own learning; I just do it				
5	I don't remember or use much from high school math				
6	I like the challenge that math offers				
7	I have a hard time staying focused, maintaining attention, keeping interest				
8	I like learning or think it is important to learn abstract stuff in math				
9	I've been out of school a long time				
10	I am not as successful in algebra as I would like because I am overloaded with other responsibilities				
11	I have a goal and I am focused				
12	I feel confident in what I know and that I can learn new material				
13	I need to feel safe enough in class to get the wrong answer				
14	The ability to solve problems is of major importance in life				
15	You should not give up				
16	I don't think I'm failing, but I don't think I'm on the honor roll, either				
17	If I can apply it to real life, it is easier to learn and remember				
18	Making mistakes is fundamental to learning; they make me aware of my own shortcomings				
19	I learn better at a steady, gentle, step-by-step, pace; don't assume that I know anything				
20	I try things over and over and do lots of problems, and then I finally get it				
21	I need to see and try things in order to understand them				
22	I learn better with group and individual work, rather than just straight lecture				
23	Reading the textbook helps to understand the material				
24	I ask for help when I need it, from people whom I trust				
25	When I solve problems myself at the chalkboard, I am more likely to remember the material; it keeps my interest				
26	I like working with partners				
27	I find technology (calculators, computers) useful for learning and gaining insight				
28	I prefer a lively paced, entertaining lecture				
29	I am always talking to myself in my head, regulating my own behavior				
30	I write in my notes everything the instructor puts on the board, because it helps me remember later				
31	I prefer to work alone				
32	Internet math tutoring tools are a great resource for learning				

	SA	A	D	SD
33 I find the solutions manual extremely useful				
34 I don't like to use technology (calculators, computers) when I learn				
35 Sitting at the front of the class forces you to pay attention				
36 I wish learning were easy and that I didn't make any mistakes				
37 When I get lost, I just try to memorize everything				
38 I prefer that the instructor takes care of all aspects of planning the course; they're the experts				
39 It's really helpful to my learning when the instructor's writing and speech are well organized, clear, and dynamic				
40 I think instructors should be flexible in their policies and their teaching methods				
41 I would like to know as much about instructors' teaching styles and policies as possible before I sign up for their classes				
42 The way professors teach and their attitudes and patience have a lot to do with student success				
43 I want my instructor to show me how to do things in algebra and explain to me why they work				
44 I like it when instructors give every student personal attention				
45 I appreciate when instructors can relate what we are learning to other disciplines				
46 What makes a bad math teacher is when he or she does not get the class to participate actively in the lesson				
47 Teachers seem to be busy, so I don't want to burden them with my questions				
48 When professors say too much about a concept, I get bored and distracted				
49 I don't like instructors to present material before the exam that isn't on the current exam				
50 Assigning homework is essential; I must do problems to learn the material				
51 Regular and timely feedback is very helpful for learning				
52 The textbook is hard to read; it's too big and too expensive				
53 Lecture is an essential part of the learning process				
54 I understand in class, but when I get home, I can't do it				
55 I prefer no exams; there's too much pressure, I'm not a good test taker				
56 Missing class is devastating to learning				
57 Sometimes I am so confused that I don't even know what to ask				
58 I don't like my entire grade to be based only on three exams and a final				
59 *Exam problems should test technical skills; they shouldn't be applied or word problems*				
60 Textbook problems are way too hard				
61 Textbook and homework problems should be much harder to prepare us for exams				
62 Some students are not prepared or serious and should not be in our class				
63 *Tests and grades based on them are clear indicators of how well you know the material*				
64 Some people need to work harder to learn instead of complaining about the class				

		SA	A	D	SD
65	Without tests, I would lack motivation				
66	Quizzes, although stressful perhaps, are good to help me keep on track				
67	The desks are way too small and too close together; it's very uncomfortable and impedes my learning				
68	Class handouts are very useful				
69	Math takes a lot of time; there's too much information				
70	I find it difficult to remember all the formulas, rules, and steps, all the tedious details				
71	Math is fun and exciting when I understand it				
72	I have a hard time getting the equations for word problems				
73	I find graphing hard				
74	I like solving equations because it's easy, concrete, like solving a puzzle				
75	Fractions are difficult				
76	I can tell when an answer doesn't look right; answers should be nice numbers without parentheses				
77	Math can be very tricky				
78	I have difficulty with radicals				
79	Reciting formulas and mimicking steps is not the same as knowing and understanding math				
80	I don't understand variables				
81	I have difficulty with complex numbers				
82	I like math; it's logical, it makes sense				
83	I think algebra should be taught much earlier than high school, perhaps even in elementary school				
84	Inequalities are difficult for me				
85	Geometry is difficult				